Atmospheric Turbulence
Its effects and what we can do about them

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Primary effect on the PSF
Adaptive optics as a compensator
Physical origin of the dynamic aberration
Effect on the propagation of an electromagnetic field
Statistical quantities of importance to imaging

The problem
(Wave propagation through a random medium)

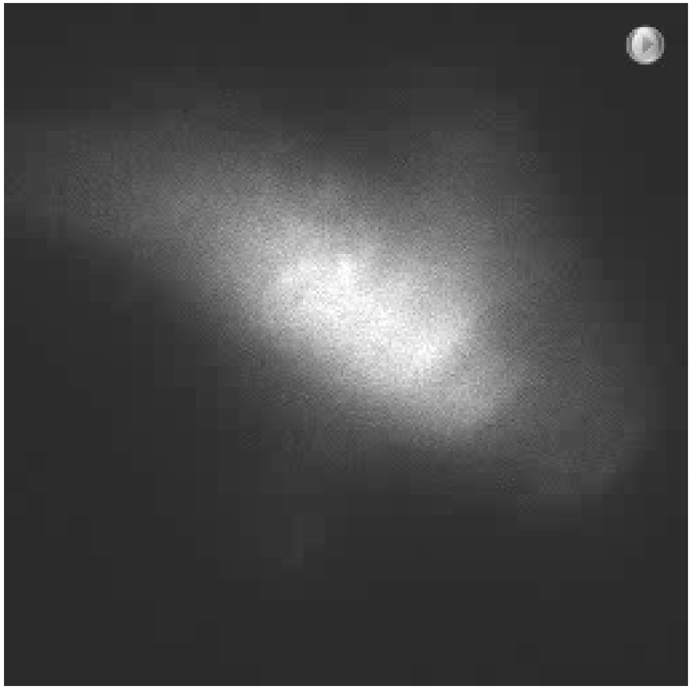
•Imaging a scene through a dynamic medium with random refractive index variations leads to loss of image quality, sometimes dramatically.

•We seek to obtain a better estimate of the object's brightness distribution by characterizing and removing the blurred point-spread function from the image.

•Image compensation can in principle be done *before detection* with adaptive optics or *after detection* numerically.

(One of) The solutions (Adaptive optics)

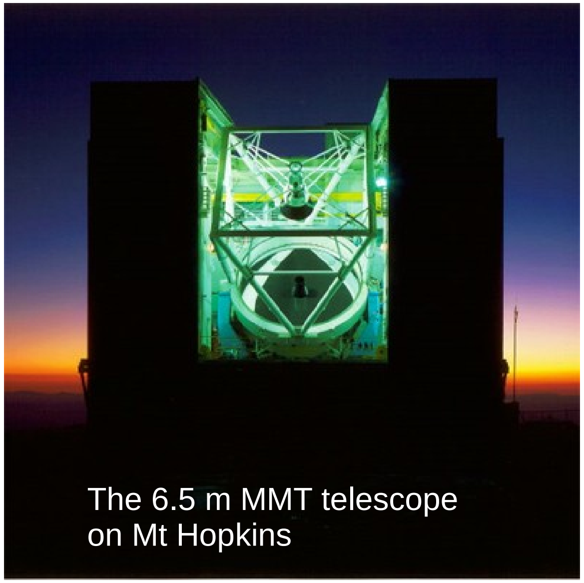
The 3.6 m AEOS telescope on Haleakala, Hawaii

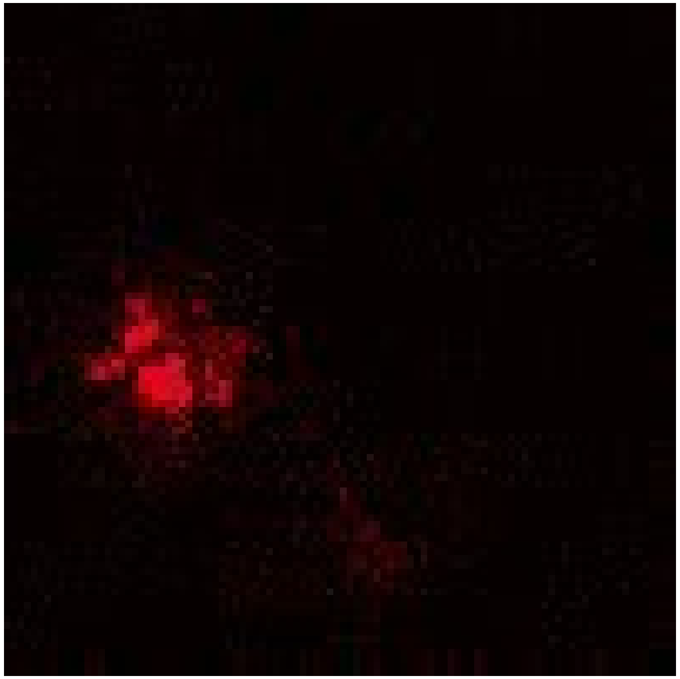


What on earth (or otherwise) is it?? AHA! Another telescope!

•The Air Force runs a 3.6 m telescope on Maui that has a high-order adaptive optics system on it.
•In this instance, they cycled the AO system on and off in real time as they tracked HST overhead.
•The resolution improves from the seeing limit imposed by atmospheric turbulence *about 1 arc second* to the diffraction limit of the 3.6 m at the wavelength of observation (750 nm) or *0.04 arc seconds*, a **factor of 25 improvement**!

Effect on the PSF





T Tauri triple star system

- Only a million years old
- Near the Hyades cluster (the head of the Bull)
- Still coalescing from the cosmic soup

•Removing the image blur improves both *resolution* and *detectability*

Adaptive Optics Applications

Astronomy
AO is a key component of current and future large astronomical telescopes.

Imaging satellites

Laser beam communication

Vision science
retinal imaging

Directed energy systems

Very Large Telescope, Chile

Truck engine melted by 30 kW laser beam

Adaptive Optics Overview

What (Good) is Adaptive Optics?

System Overview

Atmospheric Turbulence

Image Structure

Useful Relations

Nuts and Bolts of AO

Wavefront Sensors and Correctors

Natural and Artificial Guide Stars

AO Modes

AO refinements

Wide Field AO

High Contrast Imaging

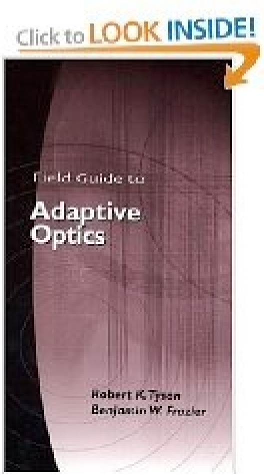
References:

Adaptive Optics for
Astronomical Telescopes

John Hardy

SPIE field guide
for Adaptive Optics

Robert Tyson



Wave Propagation & Imaging through Random Media

AO is traditionally used to improve images in “weak scattering” of light through atmospheric turbulence.

There are many other problems and scattering regimes where the same theory applies. This subject is not limited to optics and light but can be applied to radio, acoustics, seismology, mechanics, bioengineering, etc.

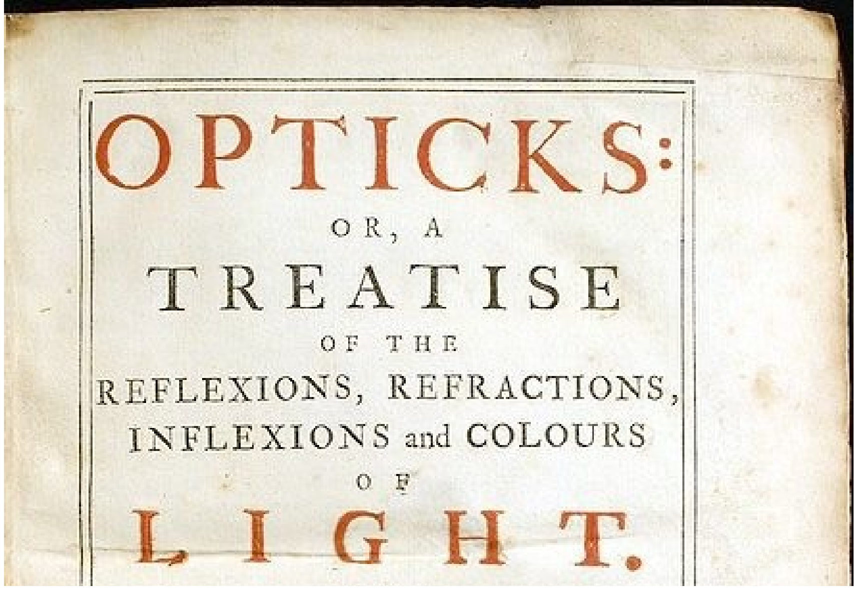
There are scattering problems where traditional AO will not work.

Even in the worst cases, a deeper understanding of the problem will enable you to make best use of current technology, and prepare you to make innovative new advances in the future.

Understanding the basic theory along with AO technology can also be used to build sensors and control systems much more advanced than are in use today.

Background

- It has been long known that the atmosphere limits the resolution achievable by optical telescopes. For example, Newton wrote



Newton, 1704

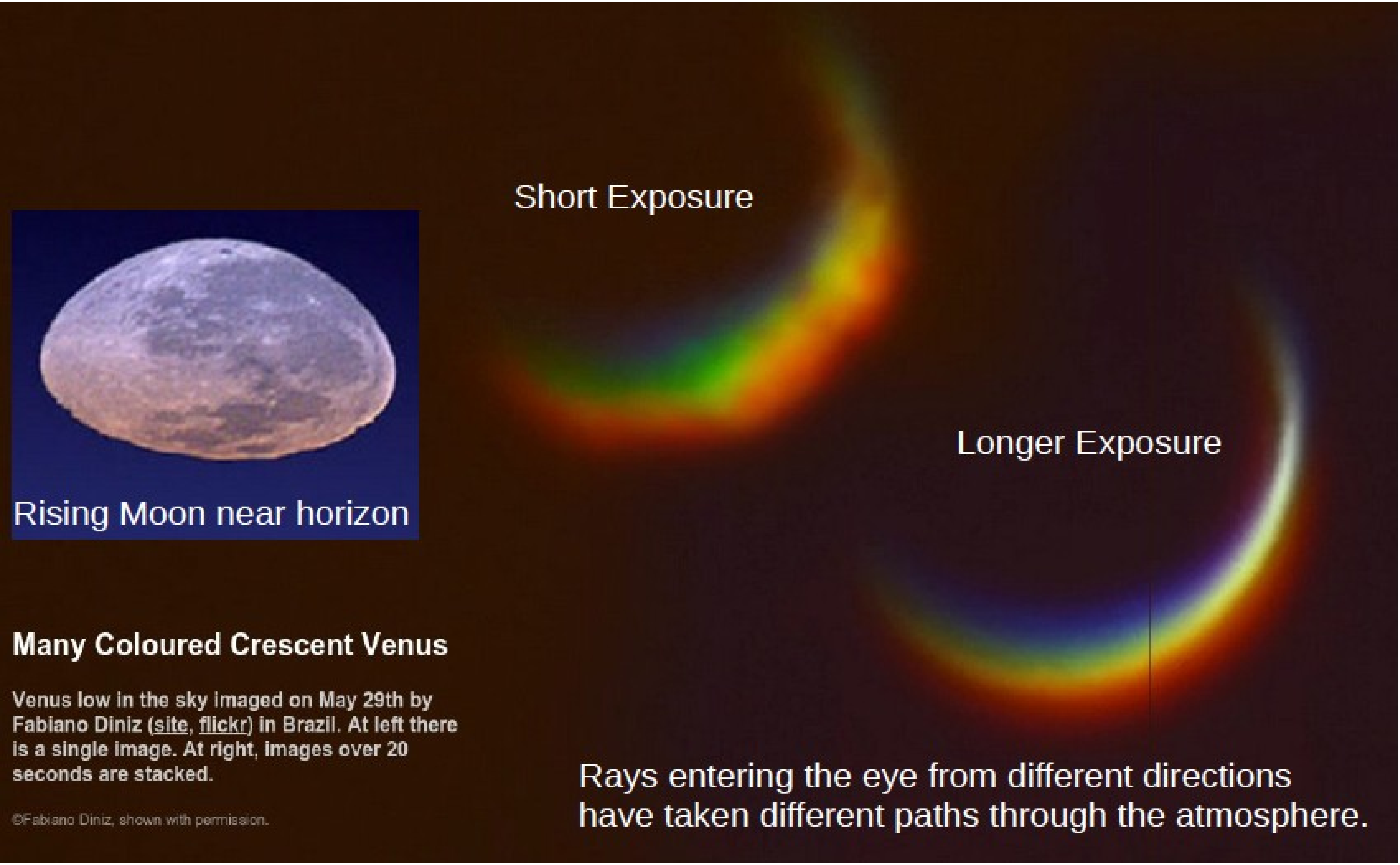
" . . . the Air through which we look upon the Stars, is in a perpetual Tremor . . . all these illuminated Points [in the focal plane] constitute one broad lucid Point, composed of these many trembling Points confusedly and insensibly mixed with one another by very short and swift Tremors and thereby cause the Star to appear broader than it is, and without any trembling of the whole. Long telescopes may cause Objects to appear brighter and larger than short ones can do, but they cannot be so formed as to take away the confusion of the Rays which arises from the Tremors of the Atmosphere. The only remedy is a most serene and quiet Air, such as may perhaps be found on the tops of the highest Mountains above the grosser Clouds.

Atmospheric effects

Atmospheric effects that upset our pristine imaging:

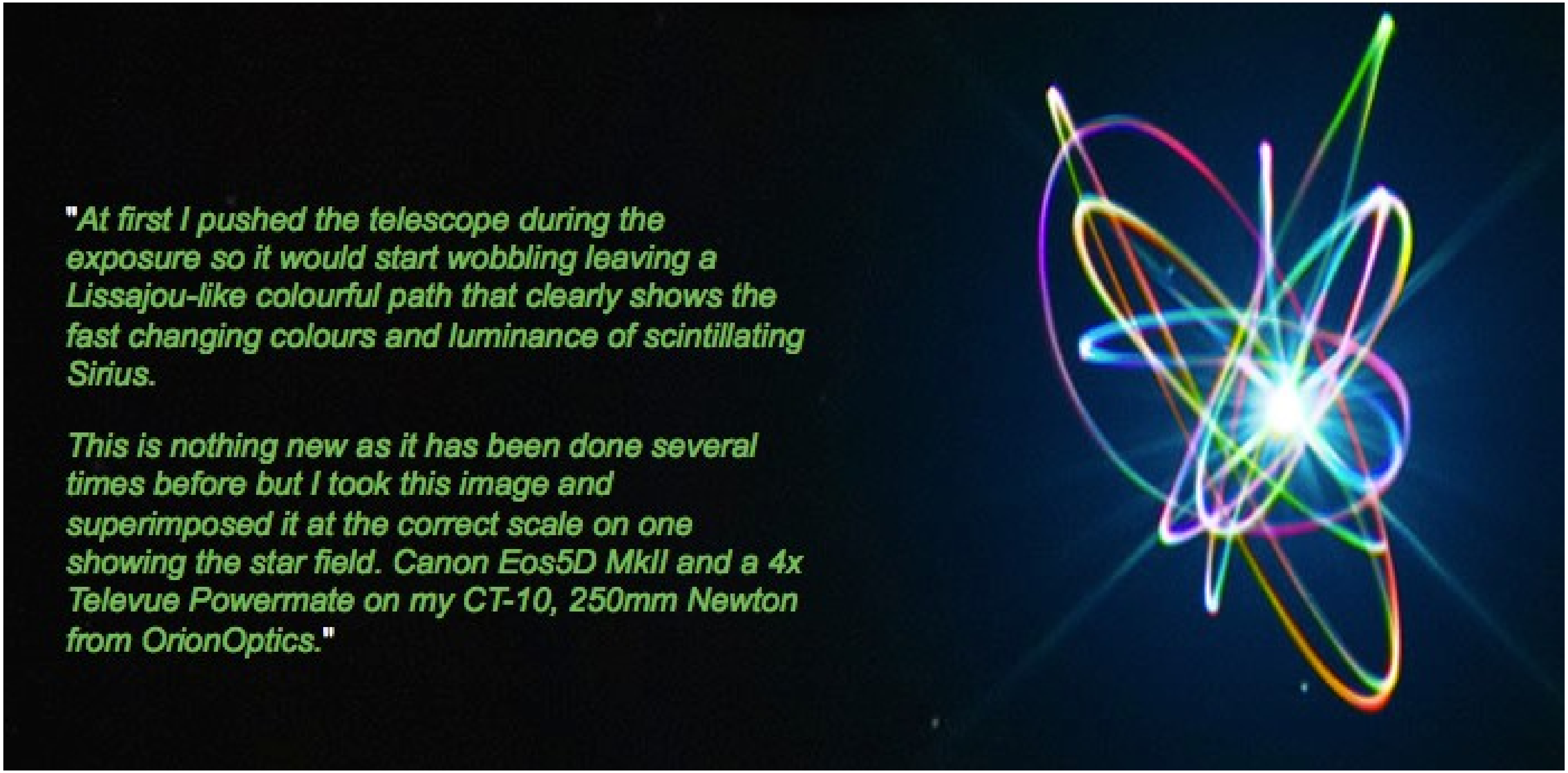
- Refraction: bending of light by air of varying refractive index
- Diffraction: light scattering by features comparable to wavelength
- Scintillation: intensity changes caused by propagation of aberrations
- Dispersion: differential refraction with wavelength that affects light paths

Refraction and dispersion



Optics Picture of the Day http://atoptics.co.uk/opod

Evolution of chromatic scintillation



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Atmospheric turbulence

- Turbulence arises when *wind shear* leads to Kelvin-Helmholtz instabilities at the boundary between shearing layers of differing density.
- Layers of air with different *density* also have different *refractive indices*, and that means light passing through gets distorted.

What is turbulence?

Turbulence is highly unstable, chaotic, nonlinear, state of fluid flow characterized by mixing. It naturally occurs when the dynamical forces of the flow far exceed the damping forces of viscosity (i.e. high Reynolds Number *Re*).

But… it is not unstructured.

- Outer scale L_0: where energy is injected, e.g. vortex shedding scale in a shear layer
- Inner scale l_0: smallest scale where energy is dissipated by viscous forces into heat
- Between these two scales is the *inertial sub-range*

Cascading scales

Energy is injected to form eddies at some large scale L_0

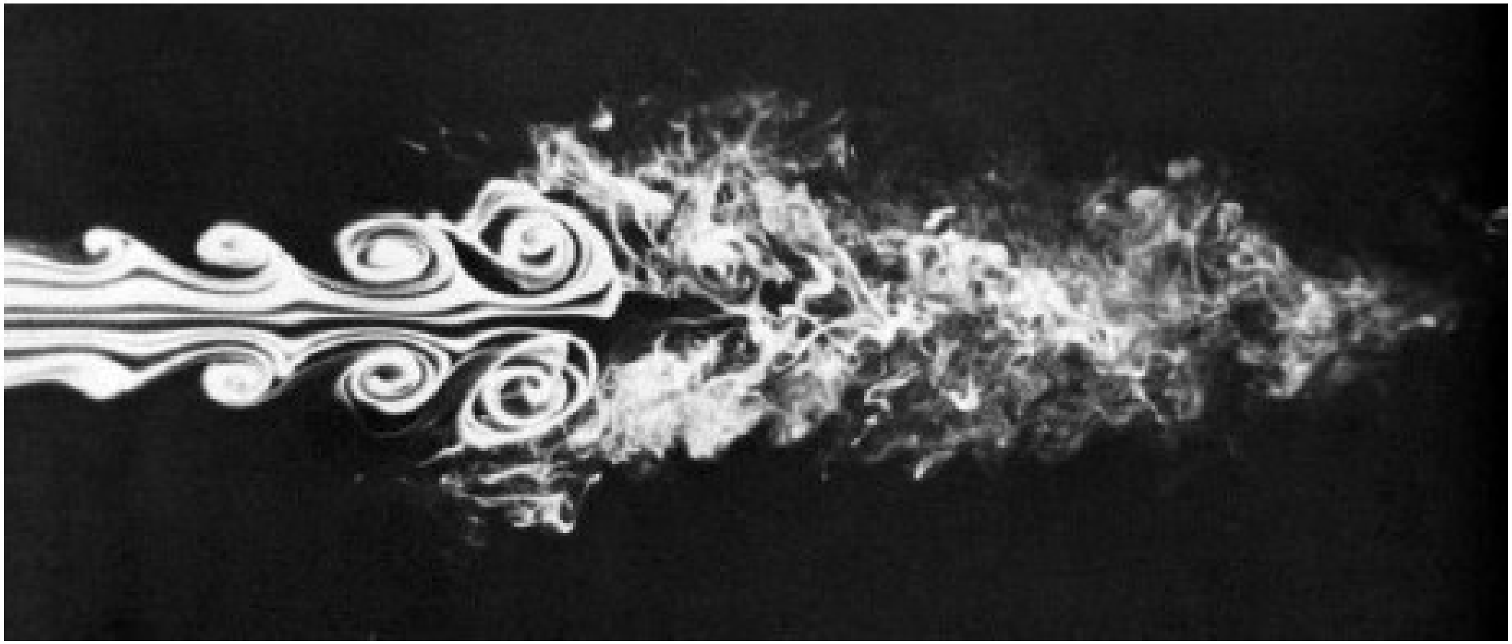
These eddies are unstable and break on some time scale into smaller eddies

This process repeats until the kinetic energy in the eddies reaches the small scale l_0

At that point, molecular diffusion takes over, and the energy is dissipated.

This cascade of energy leads to a statistically self-similar distribution of kinetic energy.

Turbulence evolving



Big whirls have little whirls that feed on their velocity;
And little whirls have lesser whirls, and so on to viscosity.

- Lewis Fry Richardson

Self similarity

In inertial sub-range, kinetic energy distribution is self-similar

So are many many other things in nature…

Self-similar kinetic energy distribution means that many other characteristics of the flow field are also self-similar, like temperature, pressure and humidity distributions – hence **refractive index**

Kolmogorov turbulence

In 1941, Andrei Kolmogorov published a theory of turbulence, which we still rely on, starting from not much more than dimensional analysis.

Three hypotheses for the case of very high *Re*:

1. Eddies on scales much smaller than L_0 are statistically independent of the components of motion at L_0 (local isotropy).

2. Statistics of velocity components on small scales are universally and uniquely determined by the viscosity ν and the rate of energy dissipation ε (first similarity).

3. Statistics of the velocity in the inertial sub-range are universally and uniquely determined by the scale k and the rate of energy dissipation ε (second similarity)

Cascading scales

On any given scale *k*, only two parameters matter

- The kinematic viscosity ν (m^2s^{-1})
- The energy dissipation rate per unit mass ε (m^2s^{-3})

From these parameters we can form characteristic time, speed and length scales

- $t_{kol} = (\nu/\varepsilon)^{1/2}$
- $v_{kol} = (\varepsilon\nu)^{1/4}$

$L_{kol} = (\nu^3/\varepsilon)^{1/4}$

Kinetic energy spectral density

Energy per unit mass per unit spatial frequency has units of m^3/s^2

Assuming a self-similar cascade, i.e. a power-law dependence on the variables, we can write the energy spectral density in terms of the characteristic scales of the problem as:

$$E(k) \alpha\ v_{kol}^2 L_{kol} (k L_{kol})^{\alpha}$$

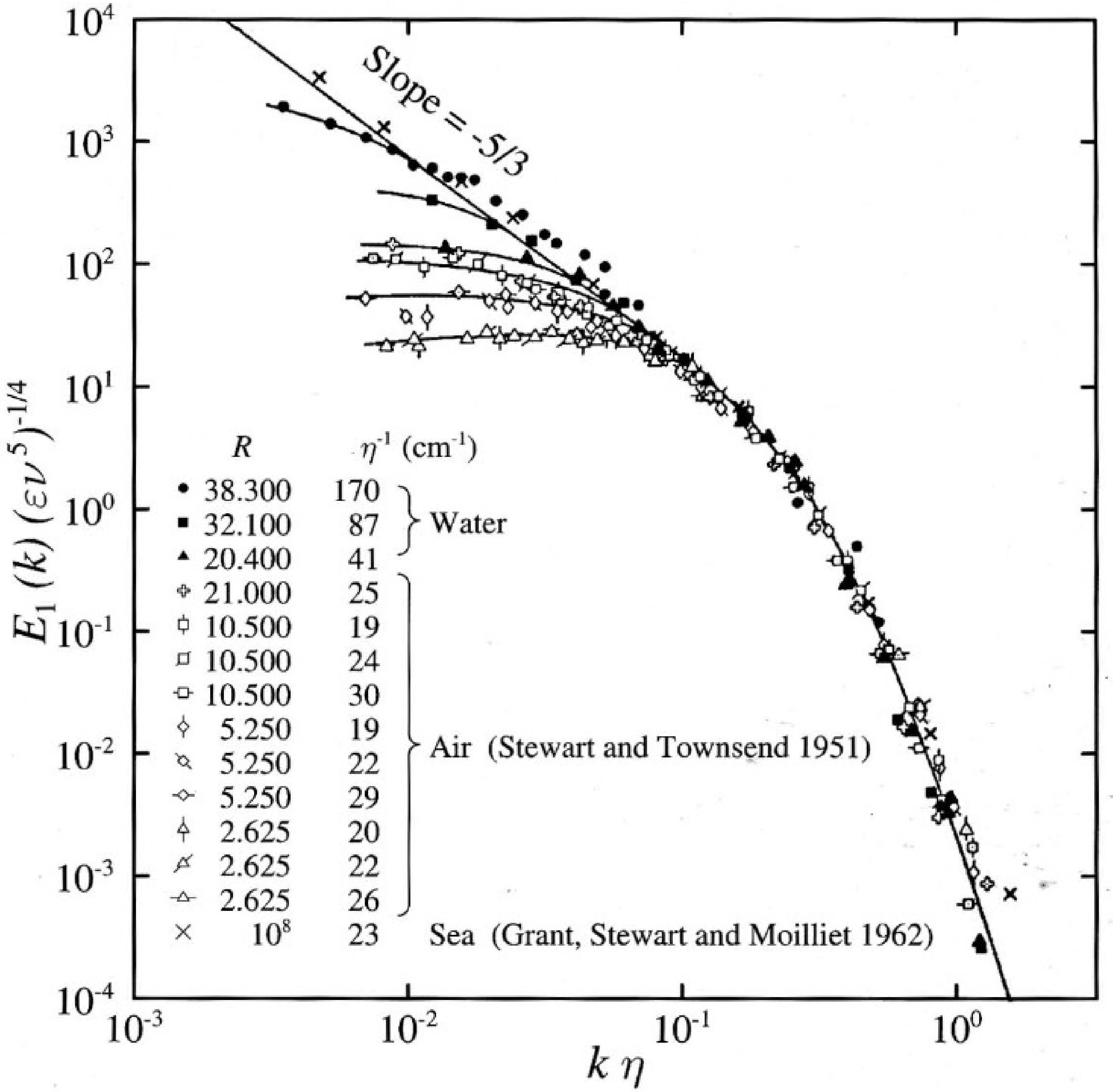
$$= \varepsilon^{(1-\alpha)/4} \nu^{(5+3\alpha)/4} k^{\alpha}$$

But, by hypothesis 3, the energy spectrum does not depend on the viscosity. So, α = -5/3, and

$$E(k) \alpha\ \varepsilon^{2/3} k^{-5/3}\ m^3/s^2$$

This is Kolmogorov's famous 5/3 power law.

Kinetic energy spectral density



Structure functions

A “structure function” describes the characteristic change in a random variable on a given scale.

E.g. for the speed *v* of some object that’s varying with time:

$$D_v(\Delta t) = \langle | v(t) - v(t+\Delta t)|^2 \rangle$$

If the speed at some time t is v(t), what is the expectation value of the *change in speed* $\langle \Delta v \rangle$ after some elapsed time Δt

$$\langle \Delta v \rangle = D_v^{1/2}$$

Velocity structure function

We can ask about the velocity field in the turbulence: how does it change (statistically) in space?

$$D_v(\Delta \boldsymbol{x}) = \langle | v(\boldsymbol{x}) - v(\boldsymbol{x}+\Delta \boldsymbol{x})|^2 \rangle$$

Here, $\boldsymbol{x}$ and $\Delta\boldsymbol{x}$ are vector quantities in 3 spatial dimensions.

By Kolmogorov hypothesis #1, we take D_T to be isotropic, which means $D_v(\Delta \boldsymbol{x}) = D_v(|\Delta \boldsymbol{x}|)$

Using the same dimensional analysis as before we can say

$$D_v(\Delta x) = C v_{kol}^2 (\Delta x / L_{kol})^a$$

Substitute for the energy density and viscosity, and insist (Kolmogorov hypothesis #3) that there is no viscosity dependence, and we arrive at a=2/3:

$$D_v(\Delta x) = C_v^2 \Delta x^{2/3}$$

C_v^2 is called the *velocity structure constant*

Effect on light waves

Refractive index of air is dependent on

- Temperature
- Pressure
- Humidity
- CO_2 content

as well as other more minor effects.

The Kolmogorov turbulent velocity field causes random fluctuations in all these quantities. Pressure is rapidly evened out by sound waves. After that, temperature matters most, and since temperature variations smooth out more slowly, that's the quantity that ties **turbulence** to **optical aberration**.

Temperature restoring time scales

Temperature variations relax more slowly than pressure variations by the usual mechanisms

- conduction (most important)
- convection
- radiation (very inefficient: air is *almost* isothermal)

Convection doesn't do much because the velocity field is completely dominated by the turbulence.

The result is that the *refractive index* follows the *temperature* which follows the *velocity field*.

Refractive index structure function

Hence, we can define a refractive index structure function and state that

$$D_n(\Delta \boldsymbol{x}) \propto D_v(\Delta \boldsymbol{x})$$

$$= C_n^2 \, \Delta x^{2/3}$$

This equation is valid for $l_0 < \Delta x < L_0$, i.e. the scale length Δx is in the *inertial subrange*

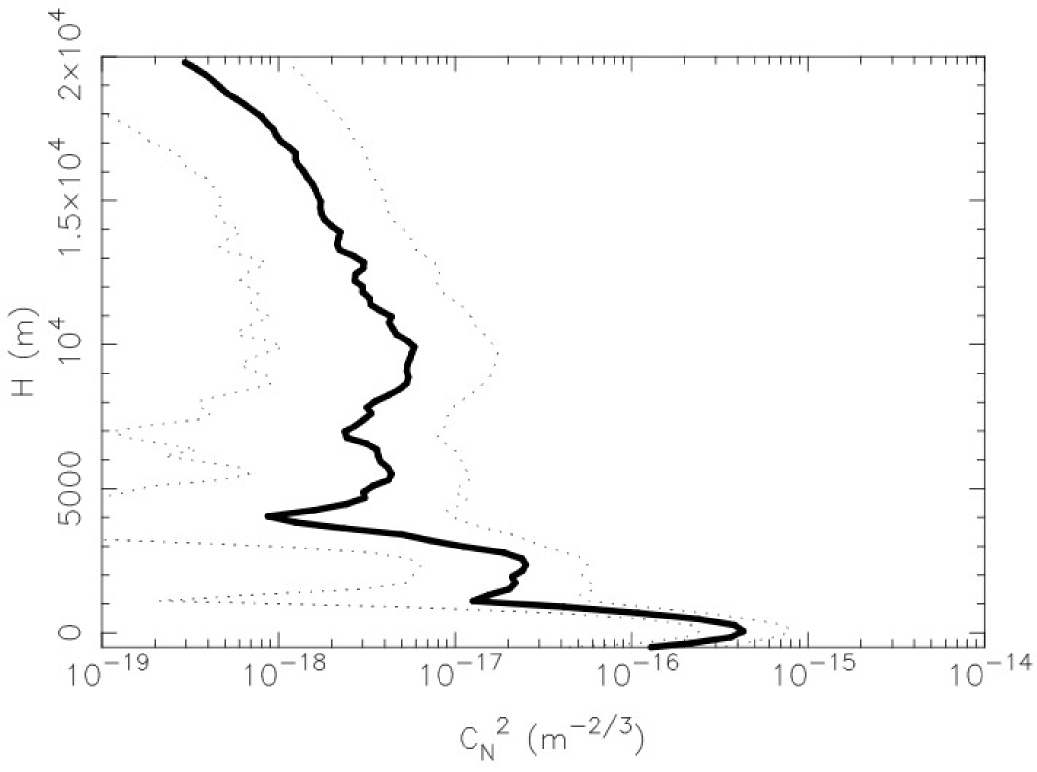
By analogy with C_v^2, C_n^2 is the refractive index structure constant. It is determined by, and is a measure of, the strength of the turbulence

It has the very peculiar units of $m^{-2/3}$

C_n^2 profiles

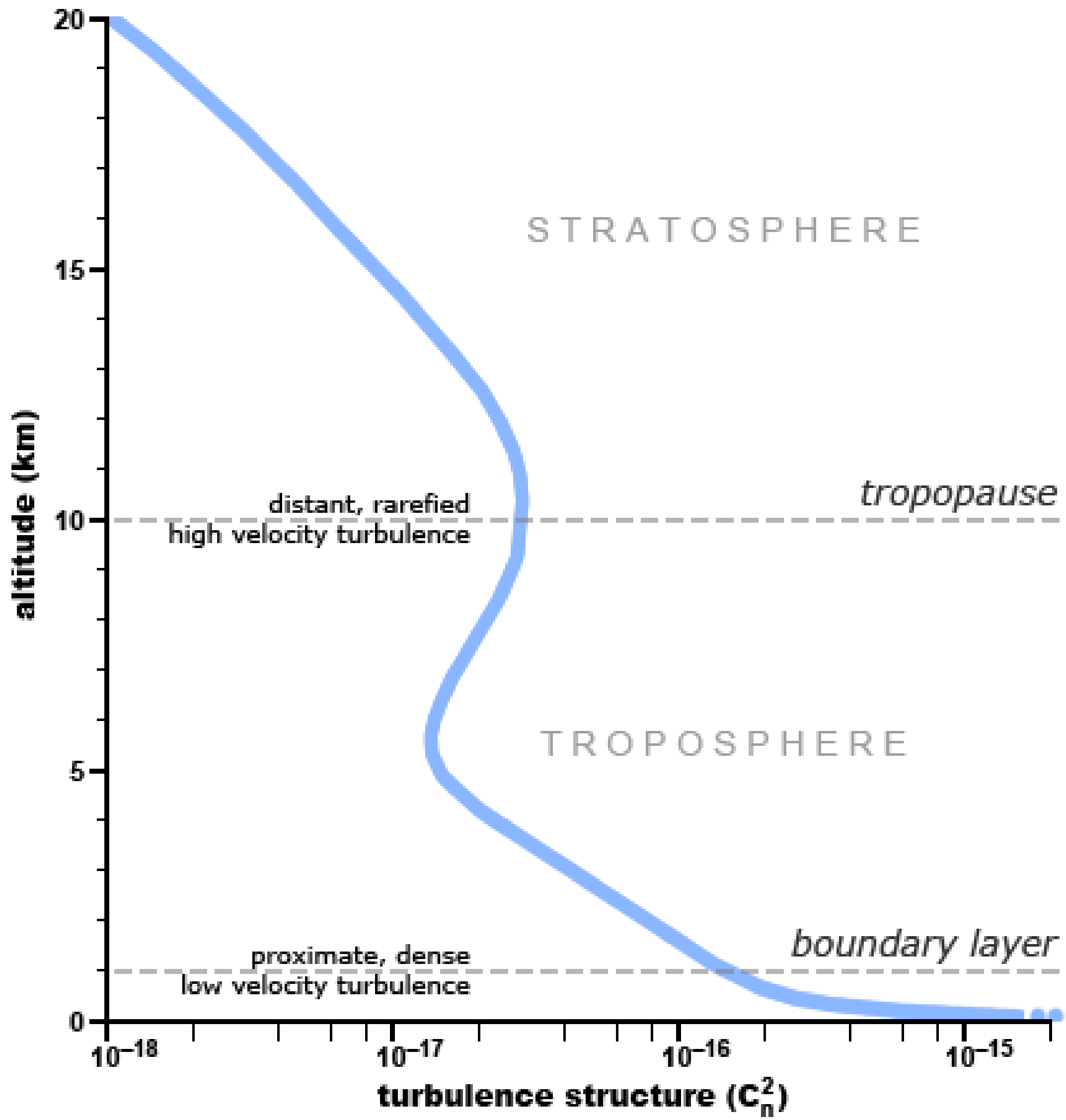
- The strength of the turbulence is measured as an index of refraction variation (termed C_n^2).

- The turbulent layers are not limited to the ground, but extend well up into the troposphere.



C_n^2 profile measured with SCIDAR at Mt. Graham
Credit: Elena Masciadri

Hufnagel-Valley C_n^2 model



$$C_n^2(h) \approx C_n^2(0)e^{-10h} + 2.7\times10^{-16}e^{-2h/3} + 5.94\times10^{-23}(v/27)^2h^{10}e^{-h}$$

v = mean wind speed (m/s)   h = height in km (not meters)

Effect on optical phase

Define the phase structure function as

$D_\phi = \langle | \phi(r) - \phi(r+\rho)|^2 \rangle$

An element of phase dϕ along a physical distance dr:

$d\phi = k\, n\, dr$ where $k = 2\pi/\lambda$

Integrating along a path through the atmosphere, we find

$D_\phi(\rho) = 6.88\, (\rho/r_0)^{5/3}$

where $$r_0 = \left[0.423\, k^2 \int_r C_n^2(z)\, dz\right]^{-3/5}$$

r_0 is the Fried length scale. We will see this a LOT in the coming weeks

Effect on optical phase

Mean square wavefront phase error over a circular aperture of diameter *D*:

$$\sigma_{\phi}^{2} = 1.0299\left(\frac{D}{r_0}\right)^{5/3}$$

σ^2 is in units of radian^2 at the wavelength of observation.

From this equation, one key physical interpretation of r_0 becomes apparent:

The Fried length is the diameter of a telescope for which the atmospheric wavefront is aberrated by ~ 1 rad rms

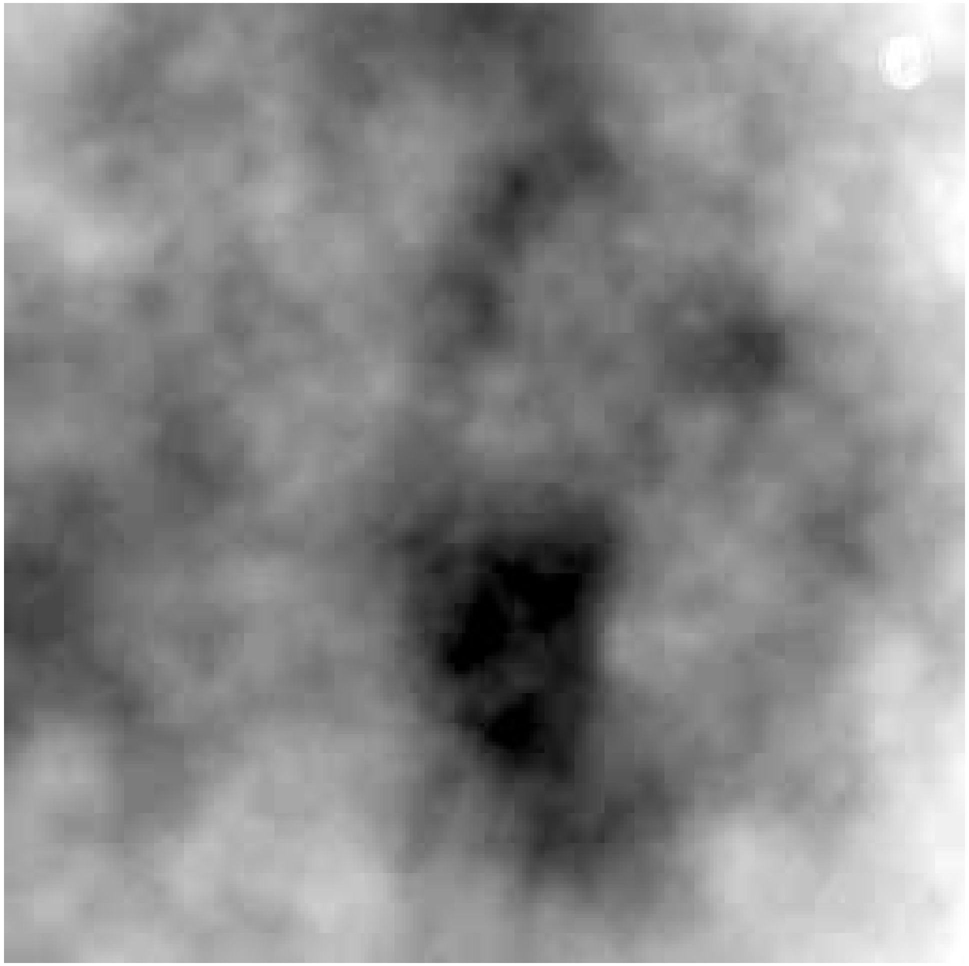
The strength of the aberration depends only on the aperture size D in units of the Fried length.

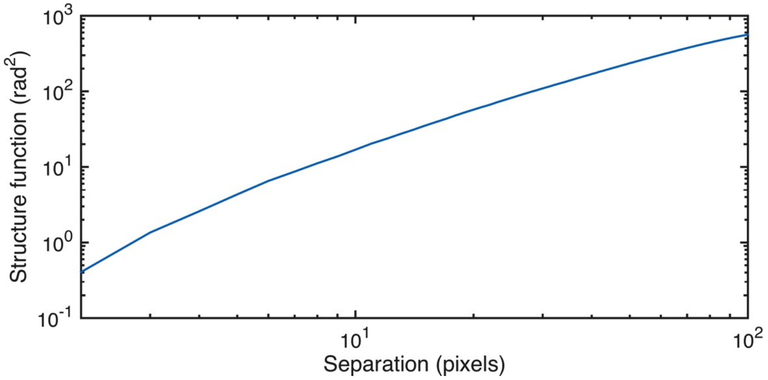
Modeled Kolmogorov turbulence

Four phase screens being blown along, each one "frozen"

A common way to model evolving atmospheric aberration, but not entirely accurate since it doesn't properly capture the temporal moments of real turbulent aberration.

There are variations going on at all spatial scales here – as expected (see the structure function plot)

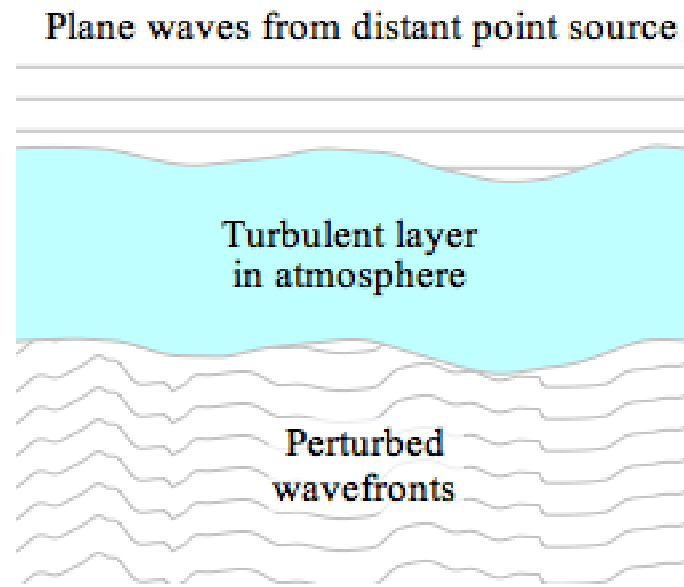


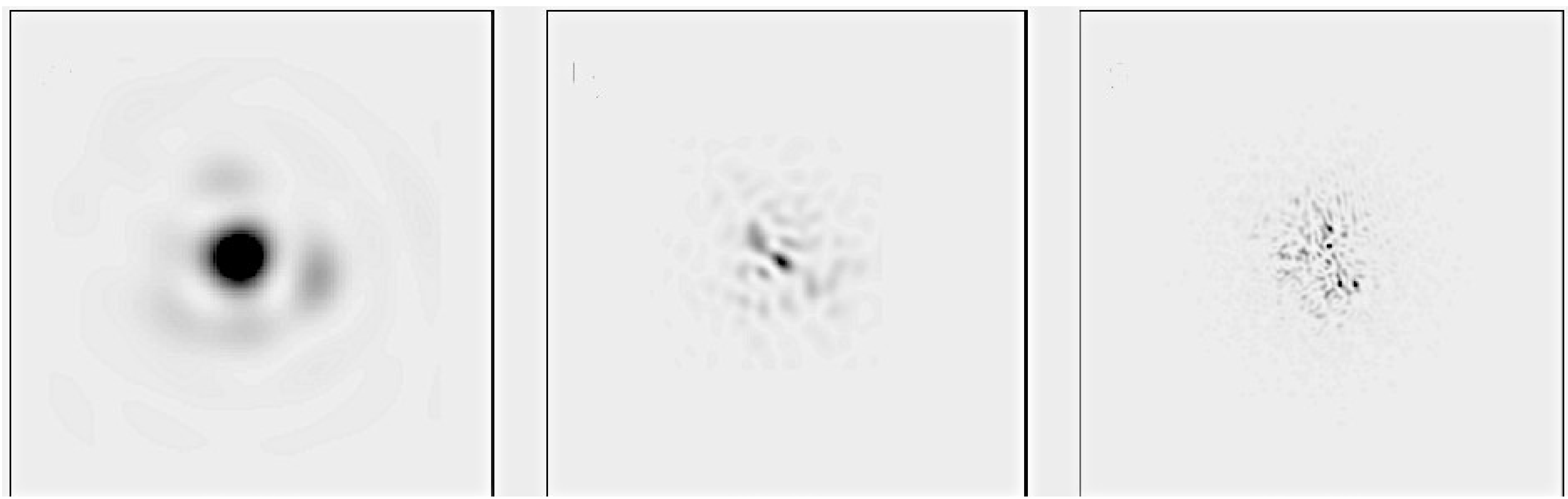


PSFs with increasing aperture

I assume you're familiar with Fourier optics.

Speak up now otherwise....





(8in telescope @ V, r_0=10 cm (30in telescope @ V, r_0=10 cm (2m telescope @ V, r_0=10 cm

Short-exposure simulated PSFs

Typical values of r_0 and seeing

Good site (e.g. Mt. Graham, Mauna Kea)		Bad site (e.g. Albuquerque)
λ(μm)	r_0 (m)	r_0 (m)
0.50	0.2	0.05
0.75	0.33	0.08
1.0	0.46	0.12
2.5	1.38	0.35
5.0	3.17	0.79
10.0	7.28	1.82

Astronomical “seeing” is defined as the FWHM of the long-exposure PSF, i.e. seeing = λ/r_0. It is almost always quoted at an industry-standard wavelength of 0.5 μm.

Coherence time

The atmospheric aberration is dynamic. If I pick a point in the pupil of my telescope, I can ask how much the optical phase changes after some time delay t:

$\sigma^2 = (t/\tau_0)^{5/3}$

τ_0 = coherence time "Greenwood time delay"= $0.314\ r_0/v$

$\tau_0 \sim$ 4 ms (visible)

25 ms (K band)

Again, τ_0 is a function of the wavelength because OPD is the thing that's changing, making for less phase change per unit time at long wavelengths than at short wavelengths.

$\tau_0 \propto \lambda^{6/5}$

Isoplanatic angle

The details of the aberration depend on the line of sight through the atmosphere

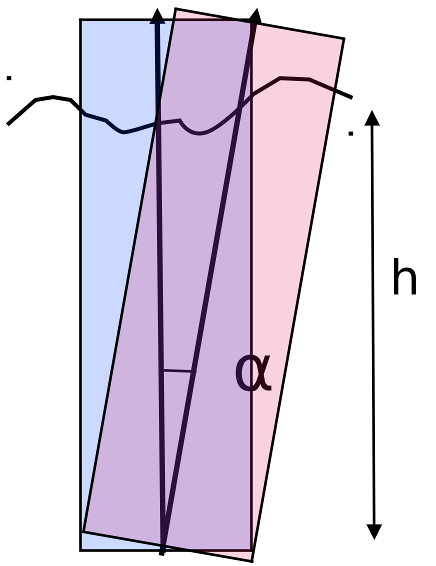
Aberration in the pupil of the telescope will be common to all lines of sight, but the overlap in beam footprints diminishes at higher altitude layers of turbulence.

$\sigma^2 = 1.03\ (\alpha/\theta_0)^{5/3}$

Where α is the angle to the optical axis,

θ_0 is the isoplanatic angle:

$\theta_0 = 0.31\ (r_0/h)$



θ_0 is also a function of wavelength: $\theta_0 \propto \lambda^{6/5}$

Scintillation

2mm/pixel, 1024x1024 pix (~2m x 2m)
lambda=500nm, 30 deg zenith angle, 0.8” seeing at zenith
Site: Mauna Loa observatory (3500 m altitude)