Direct imaging: the natural choice for cophased interferometry

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Abstract: Future interferometric instrumentation mainly relies on the availability of an efficient cophasing system. The transition to come from coherenced to cophased interferometry is very comparable to the one we've already lived through from speckle interferometry to adaptive optics imaging with a single dish telescope. Among the possible uses of such a cophased interferometer, global beam combination for direct imaging appears like one of the simplest and most elegant solutions. To make direct imaging possible, the hypertelescope combination scheme is definitely the most appropriate. Indeed, far from inducing field loss, as it is often reproached, a well chosen pupil densification proves to be the optimal use the collected photons: it simply reduces direct imaging field and makes it equal to the only field accessible to the array itself.

1 Introduction

Ambitious projects of interferometric arrays (c.f. these conference proceedings) involve a large number of apertures in order to map and study complex fields of view or detect fainter sources than what is currently accessible. It seems that the most exciting astrophysical observations will forever lie at the edge of detectability: an efficient cophasing system will push back the present frontiers of optical interferometry and open new fields of invesitgation, such as coronography or spectral analysis of faint extragalactic sources.

Yet, once the interferometer cophased, direct imaging appears not only like the most elegant but also the most natural of its uses. The figure 1 compares the development of high angular resolution techniques for both interferometry and single dish telescopes: the possible use of coronogragraphy and its application for exoplanet imaging is strongly in direct imaging's favor.

2 Towards direct imaging

The two top-level requirements for providing direct exploitable images from an interferometer are a global recombination scheme and an optical image reconstruction method.

The practical advantages of a global (also called "all-to-one") over a "pairwise" beam combination are obvious when the number n_T of telescopes is large. The complexity of a pairwise combiner scales as n_T^2, while the complexity of a global combiner is not dependant on n_T.

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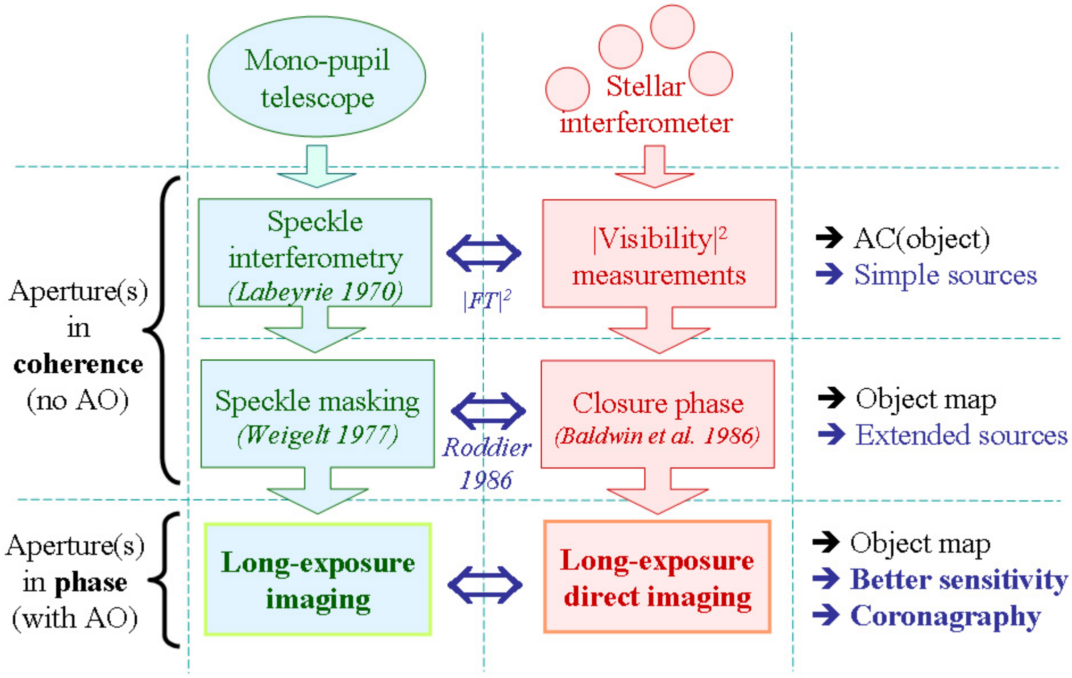

Figure 1: Evolution and analogies of high angular resolution techniques used on single telescopes and in stellar interferometry: Speckle interferometry (Labeyrie, 1970) and speckle masking (Weigelt, 1987) have been used on single telescope to recover a part of the information lost in the turbulence. Now, long-exposure imaging with adaptive optic systems is preferred to have a better sensitivity and to allow coronagraphy. Stellar interferometers know the same evolution: visibility modulus measurements and closure phase are equivalent to the speckle interferometry and the speckle masking respectively (Roddier, 1986). Then, direct imaging on cophased arrays is clearly the natural next step for stellar interferometry.

Moreover, a comparative study made by Prasad & Kulkarni (1989) shows that the senstivity of the both combinations are comparable.

Yet, even if a “all-to-one” combination scheme is retained, an optical image reconstruction method will drastically improve the quality and the luminosity of the images: this is where the hypertelescope comes. For a diluted aperture, when the combiner provides an exit pupil purely homothetic to the array itself, which is usually called the Fizeau scheme, the associated PSF spreads over a wide envelope (whose dimension’s inversely proportional to the diameter of a subaperture) into numerous speckles, therefore providing poorly contrasted images.

To overcome this weakness, Labeyrie (1996) proposes the *hypertelescope* concept. It uses the technique of *pupil densification*, that concentrates the flux over a smaller but more intense halo, drastically increasing the contrast and the SNR (if limited by the detector read-out noise) (see for instance the fig. 2). High contrast imaging and coronagraphy now becomes possible and with it, the detection and characterisation of extrasolar planets with an interferometer.

3 Fields of view in interferometry

One however often hears that the luminosity gain provided by a hypertelescope comes at the expense of a strong reduction of the field of view: it is our intent to demonstrate here that this is simply wrong.

There are *a priori* two possible sources of field of view limitation: the array itself that

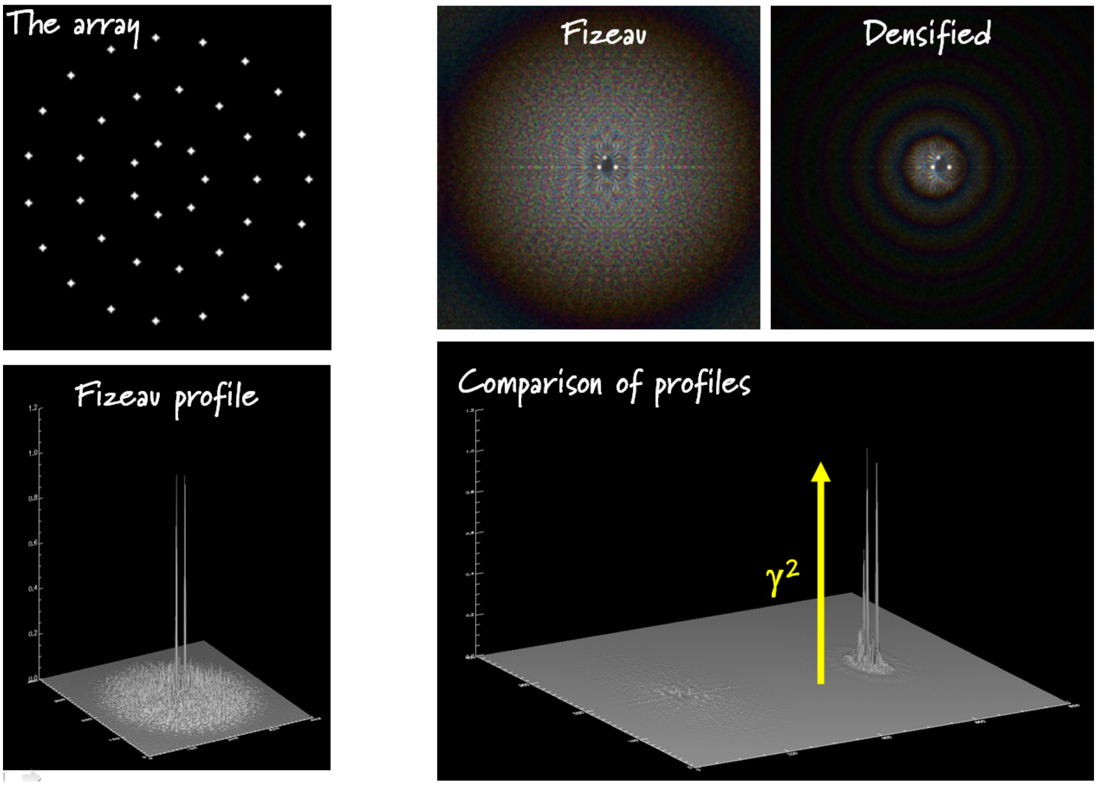

Figure 2: Direct imaging of a triple star obtained with a 39-apertures array in the Fizeau mode and in the densified pupil mode. Compared to the Fizeau, the intensity gain of the central peak is γ^2, with the pupil densification factor $\gamma = (d'/D')/(d/D)$, where d and D are respectively the diameter of a single telescope and of the whole array, while d' and D' are their alter ego in the output pupil plane. This gain can reach 10^4 for kilometric arrays.

offers a limited coverage of the u-v plan and the beam combiner that may or may not destroy information. This therefore forces us to define several different fields of view, whose properties will be detailed in the following paragraphs:

- The "Interferometric Field" (IF) whose maximum extent is imposed by the number of independant baselines offered by the geometry of the array.
- The "Direct Imaging Field" (DIF) is simply the field over which direct imaging of a source is possible. This field is of course only function of the beam combination scheme.
- The "Coupled Field" (CF) is the field defined by the fraction of the sky that will see at least a part of its light on the central pixel of the detector, for better or for worse. This field also depends on the retained beam combiner.

3.1 Interferometric field (IF)

Contrarily to a single dish telescope, a diluted interferometer only provides a discrete and very partial coverage of the u-v plan. Simply relying on the Nyquist-Shannon sampling theorem, a well-known rule coming from radio-astronomy reminds that: "The mappable field of view of an interferometer is given by the largest typical hole in the (u,v) coverage", i.e the minimum distance between telescopes. An illustration of the clean part of the PSF (direct or pairwise-synthesized) is given at the figure 3.

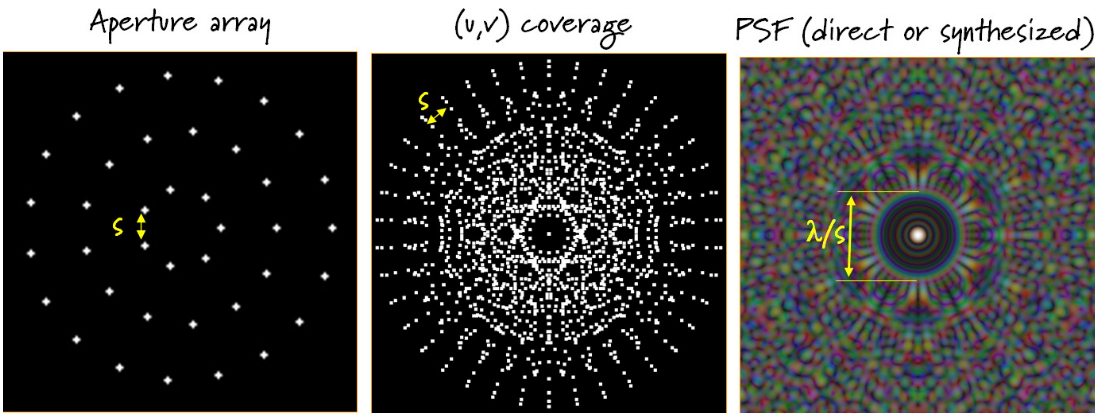

Figure 3: The clean field of an interferometric PSF is given by λ/s where s is the minimal distance between the telescopes. s is also the typical largest gap of the (u,v) coverage.

An upper limit of the IF can be found using the information theory of Shannon (1948). Without ever considerating a combination scheme, Koechlin (2003) demonstrates that for an interferometer, the ratio field/resolution is only constrained by the number of telescopes involved in one observation and by the dynamic range of measurements. One can invoke on the one hand, Shannon's entropy collected in an observation:

$$H \leq n_{obs}\ log_2(\delta_{data}), \tag{1}$$

where n_{obs} is the number of observables and δ_{data} the dynamic range of measurements. A non-redundant cophased interferometer involving n_T telescopes gives $n_B = n_T \times (n_T - 1)/2$ complex visibilities, that is $n_{obs} = 2 \times n_B$ (modulus and phase).

On the other hand, the entropy of the image reconstructed from these interferometric data is given by:

$$H_{ima} = n_r\ log_2(\delta_r), \tag{2}$$

where n_r and δ_r are respectively the number and the dynamic-range of resels (resolution elements) in the image.

In the most favorable case, these two entropy terms can be made equal. This therefore constrains the number of resels in one image as $n_r \leq n_T \times (n_T - 1)$. Since the angular size of one resel is λ/B (with B the maximum baseline), the extent of this interferometric field is given by:

$$IF \leq \lambda/B \times \sqrt{n_T \times (n_T - 1)}. \tag{3}$$

This is a very strong field limitation that is absolutely independent on the beam combination scheme: Pairwise, Fizeau, Hypertelescope or IRAN (Vakili *et al.*, 2004).

3.2 Direct imaging field (DIF)

The DIF depends only on the beam combination scheme. In case of the Densified Pupil, the DIF is given by:

$$DIF = \frac{\lambda}{(\gamma - 1)\ d} \tag{4}$$

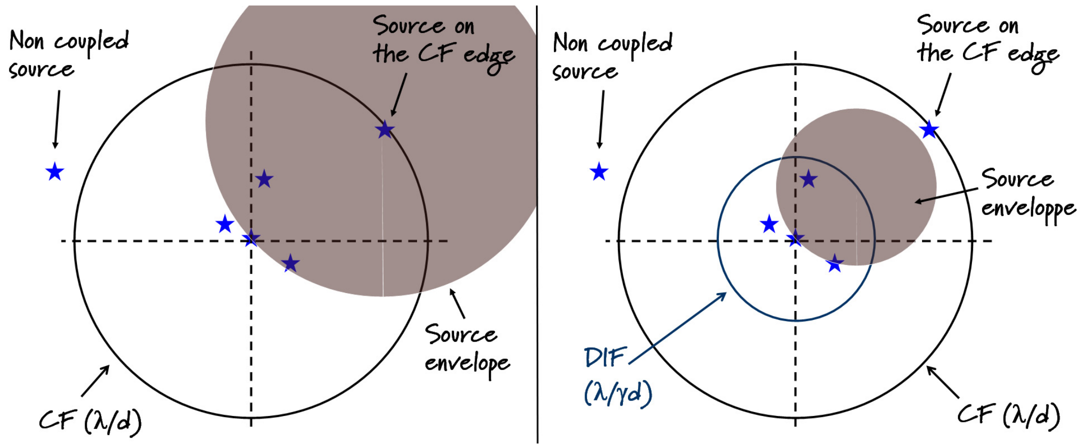

Figure 4: Coupled field of an interferometer in the Fizeau mode (left) and in the Densified pupil mode (right). The CF is the same in the both cases.

where γ is the pupil densification factor as defined in figure 2, and d the diameter of a single telescope. It is this apparent reduction of the DIF (infinite in the Fizeau case for which $\gamma = 1$) by the factor $\gamma - 1$ that leads to the conclusion that the Hypertelescope induces field loss.

The densification coefficient is however a free parameter that we can use to merge the DIF and the IF (Martinache (2005)). The equality of those two fields leads to the value of an optimal densification coefficient:

$$\gamma_{opt} = 1 + \frac{B}{d\sqrt{n_T \times (n_T - 1)}} \approx \frac{B}{d \times n_T}. \tag{5}$$

This coefficient can definitely be said optimal since it concentrates the flux as much as possible, without destroying any information: it preserves the entropy.

For an "in-line" redundant interferometer, this optimal densification is achieved when sub-apertures are made edge to edge in the output pupil, exactly as Labeyrie initially proposed, without any consideration for the field of view. This result can indeed be intuited when one realizes that when the densification that merges the DIF to the clean part of the interferometric PSF (c.f. fig. 3) is for $\gamma = s/d$, meaning that the sub-pupils in the output pupil have to be edge to edge.

It is now clear that the Hypertelescope induces no field loss: admittedly, the direct field is limited... but to the only accessible field.

3.3 Coupled field (CF) and crowding limit

One can define the coupled field that way: if a source belongs to the CF, its light will appear on the final detector. The notion of CF is essential in interferometry, for the extend of the CF decides whether or not the interferometer can obtain an image of an object from a given area of the sky.

Defining the IF, we've indeed seen that the lacunar coverage of the u-v plan by the interferometer only provides limited information. This also imposes the maximum acceptable complexity of a target field, which is known as the *crowding limit* of the interferometer.

Each source produces a central peak resulting from n_T coherent contributions and a halo resulting from the same number of incoherent contributions: the peak intensity scales as n_T^2, while the average level of the halo scales as n_T. Peaks are detectable if they dominate the fluctuations of the halo, which is $\sqrt{p}\, n_T$ with p the number of coupled sources. Then we have to meet the condition $n_T \geq \sqrt{p}\, n_T$, meaning that $p \leq n_T^2$: the number of coupled sources must be kept lower than the number of baselines. If the complexity of a source is greater than this crowding limit, one needs *a priori* knowledge on the source to be able reconstruct an image.

The figure 4 shows that there is no difference between Fizeau and Densified pupil: $CF = \lambda/d$. A difference appears when the interferences are recorded in a pupil plane like proposed with IRAN: the CF becomes infinite and the crowding becomes a serious issue. The use of a spatial filtering (pinhole or single mode fiber) to circumvent this limit makes IRAN and Hypertelescope become identical solutions.

4 Conclusions

Direct imaging with a cophased array appears as the next step that will follow visibility measurements with coherenced optics. Among the available possibilities, we've focused on the properties of the Hypertelescope concept.

Indeed, in the presence of sky background and with a non photon limited detector, pupil densification makes possible the imaging of objects too faint for other combination process by magnifying up to 10^4 times the peak intensity.

The introduction of several notions of field of view shows that contrarily to what is generally accepted, Laberie's pupil densification technique induces no field loss. It now appears as an optical image reconstruction technique that makes an optimal use of photons.

In order to progress towards direct imaging, two projects are proposed: VIDA, a hypertelescope precursor using the VLTI infrastructure (Lardière *et al.*, 2005), and Carlina, a fully-fledged hypertelescope involving dozens of small mirrors (Borkowski *et al.*, 2005).

References

Borkowski, V. *et al.*, 2005, these proceedings

Koechlin, L., 2003, EAS pub. series, 8, 349

Labeyrie, A., 1970, A&A, 6, 85.

Labeyrie, A., 1996, A&A Suppl., 118, 517

Lardière, O. *et al.*, 2005, these proceedings

Martinache, F., 2005, PhD Dissertation

Prasad & Kulkarni, 1989, JOSA, 6, 1702

Roddier, F., 1986, Optics Communications, 60, 145

Shannon, 1948

Vakili, F., Aristidi, E., Abe, L., Lopez, B., 2004, A&A, 421, 147

Weigelt, G. P., 1977, Optics Communications, 21, 55