Simulations of Baryon Acoustic Oscillations

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Covariance matrix of the matter power spectrum

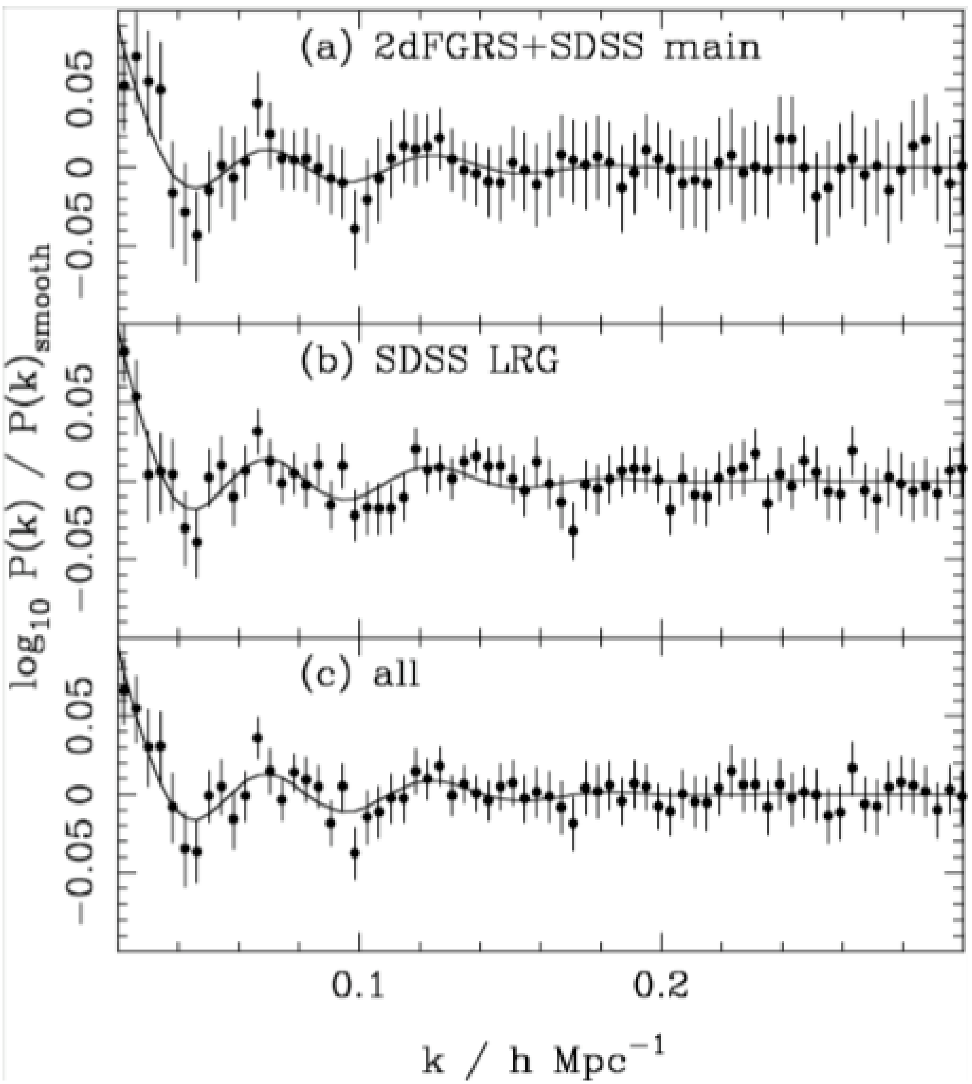
Why accurate power spectrum ?
Why covariance matrix ?

Future observations aim at extremely accurate (<%) measurements of the BAO scale.

The same level of, or even better, accuracy is required to theoretical predictions.

Covariance matrix is the key element in the Fisher Matrix analysis for cosmological parameter estimation.

Baryon Acoustic Oscillations



Current status

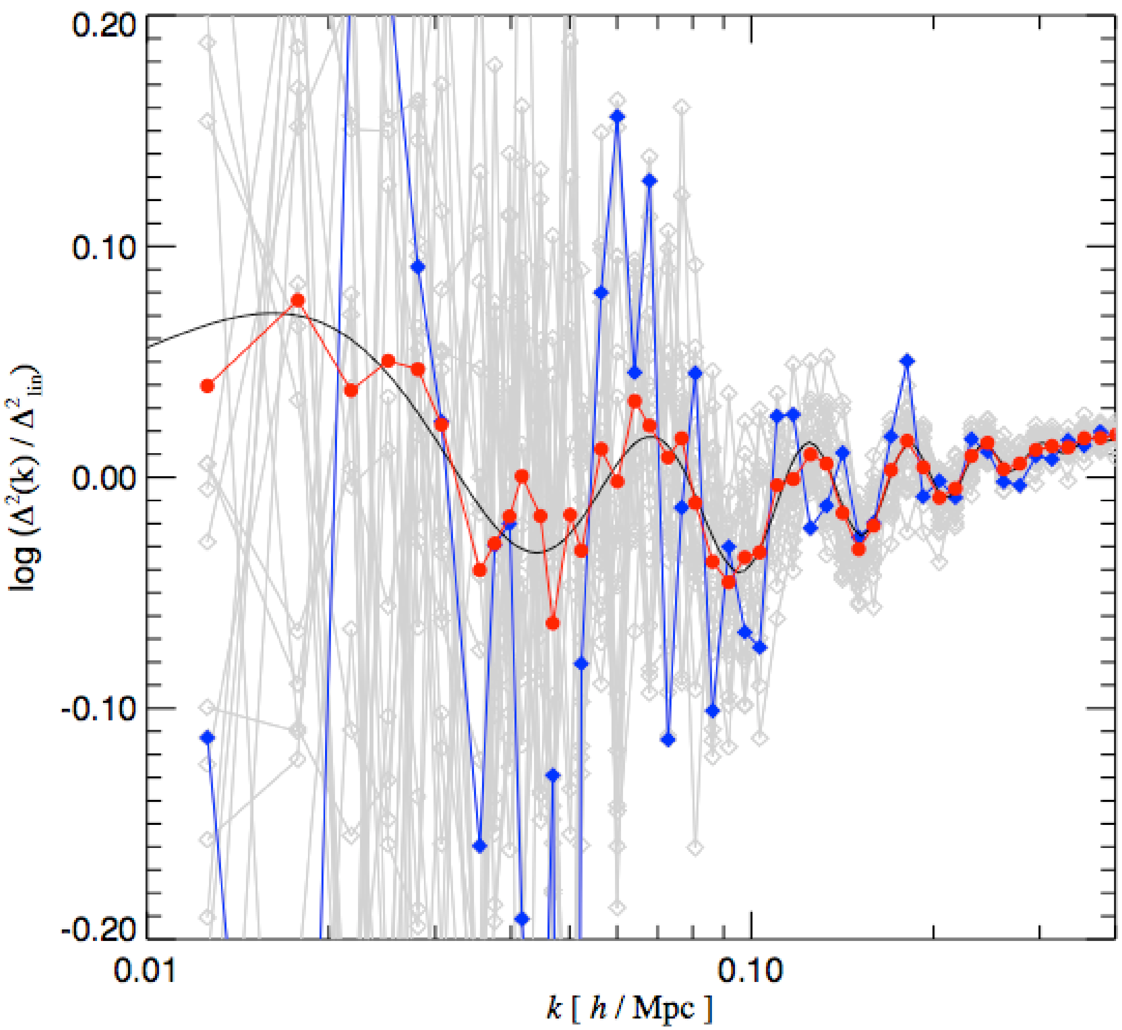
2dF, SDSS

galaxy power spectrum

Percival et al. (2007)

BAO in a Gaussian Random Field

Normalized power spectrum

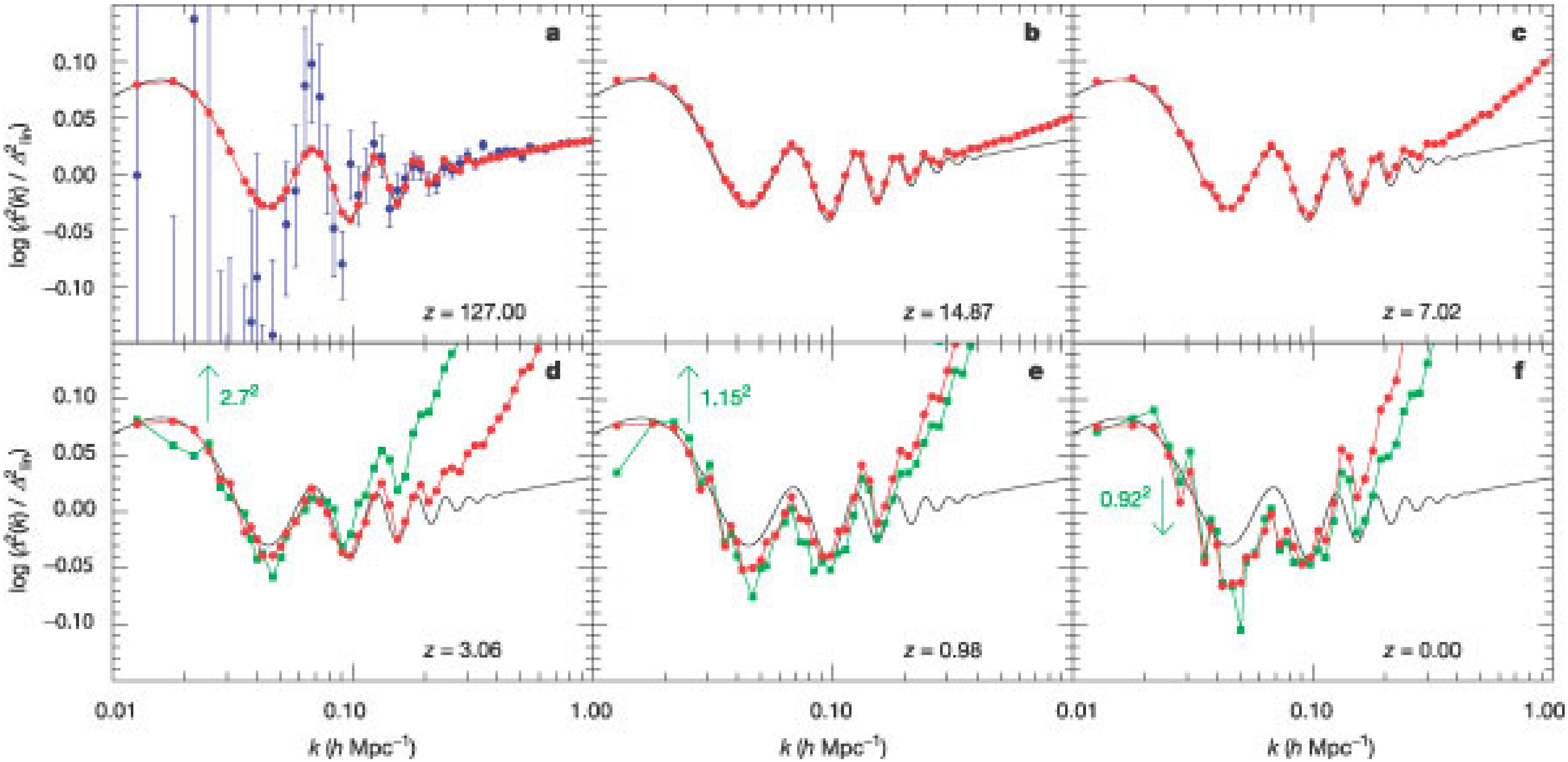


Substantial scatter around the mean expected value in a finite volume (realization)

A large volume or an ensemble of many regions (realizations) needed.

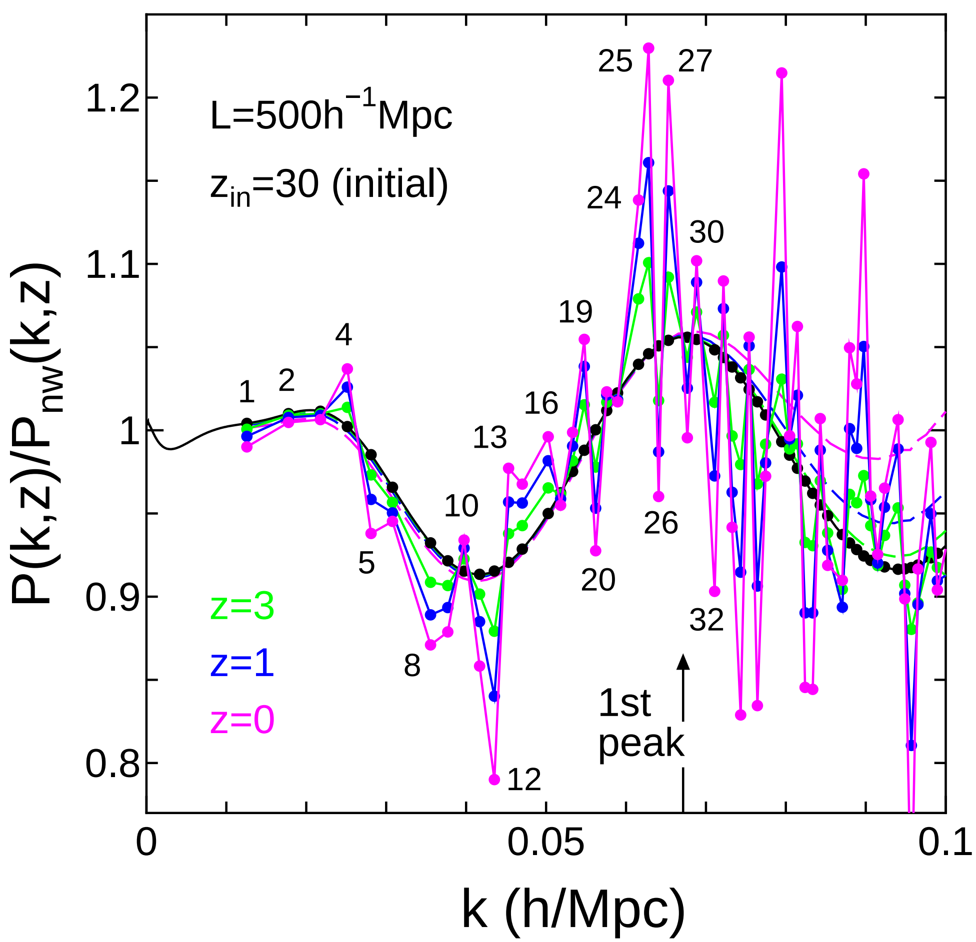
Wiggles on Wiggles

A most recent example from Millennium Simulation



"zig-zag" patterns compromise measurement of BAO

A mode-by-mode analysis



Takahashi, NY+ (2008)

An example (realization)

L=500Mpc/h

256^3 particles

$$\frac{P(k,z)}{P(k,z=30)}\frac{P_{\rm input}(k)}{P_{\rm nw}(k)}$$

$$k=\frac{2\pi}{L}n$$

$$=\frac{2\pi}{L}\sqrt{n_1^2+n_2^2+n_3^2}$$

~ 10-20% deviation from the linear perturbation theory

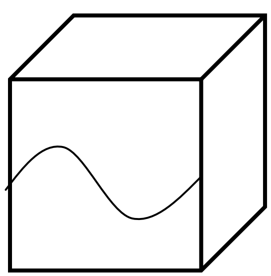
Second-order perturbation theory

$$\delta(\mathbf{k}, z) = D(z)\,\delta_1(\mathbf{k}) + D^2(z)\,\delta_2(\mathbf{k})$$

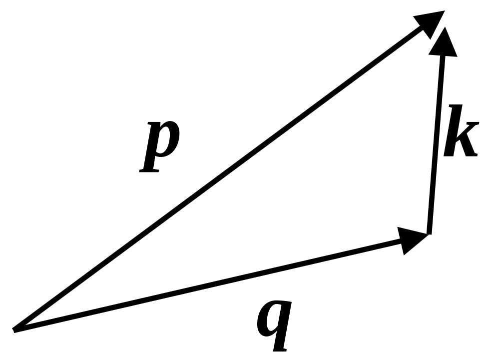
$$\delta_2(\mathbf{k}) = \sum_{\mathbf{p}} F_2(\mathbf{p}, \mathbf{k}-\mathbf{p})\underline{\delta_1(\mathbf{p})\delta_1(\mathbf{k}-\mathbf{p})}$$

$$\left|\delta(\mathbf{k}, z)\right|^2 = D^2(z)\left|\delta_1(\mathbf{k})\right|^2 + 2D^3(z)\,\mathrm{Re}\left[\delta_1(\mathbf{k})\delta_2^*(\mathbf{k})\right]$$

$P(k, z)$



K-mode vector coupling



To lowest order,
the evolution of a mode ***k***
is governed primarily by
a certain (limited) number of
particular sets of (***p, q***).

Recall the characteristics of random Gaussian fields…

each mode has a finite possibility of having a very large/small amplitude w.r.t. the mean.

Large δ(p) and δ(q) (by chance) cause "extra" growth of δ(k)

Second-order perturbation

$$\delta(\mathbf{k},z) = D(z)\delta_1(\mathbf{k}) + D^2(z)\delta_2(\mathbf{k})$$

$$\delta_2(\mathbf{k}) = \sum_{\mathbf{p}} F_2(\mathbf{p},\mathbf{k}-\mathbf{p})\delta_1(\mathbf{p})\delta_1(\mathbf{k}-\mathbf{p})$$

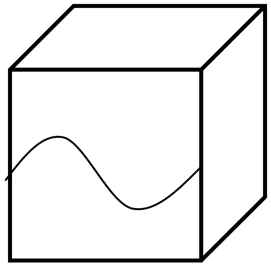
δ_1, δ_2 ←**directly measured from the initial density fluctuations**

$$|\delta(\mathbf{k},z)|^2 = D^2(z)|\delta_1(\mathbf{k})|^2 + 2D^3(z)\,\mathrm{Re}\left[\delta_1(\mathbf{k})\delta_2^*(\mathbf{k})\right] \leftarrow \mathrm{O}(\delta^3)$$

↑

$P(k,z)$

This term should vanish by ensemble-averaging.

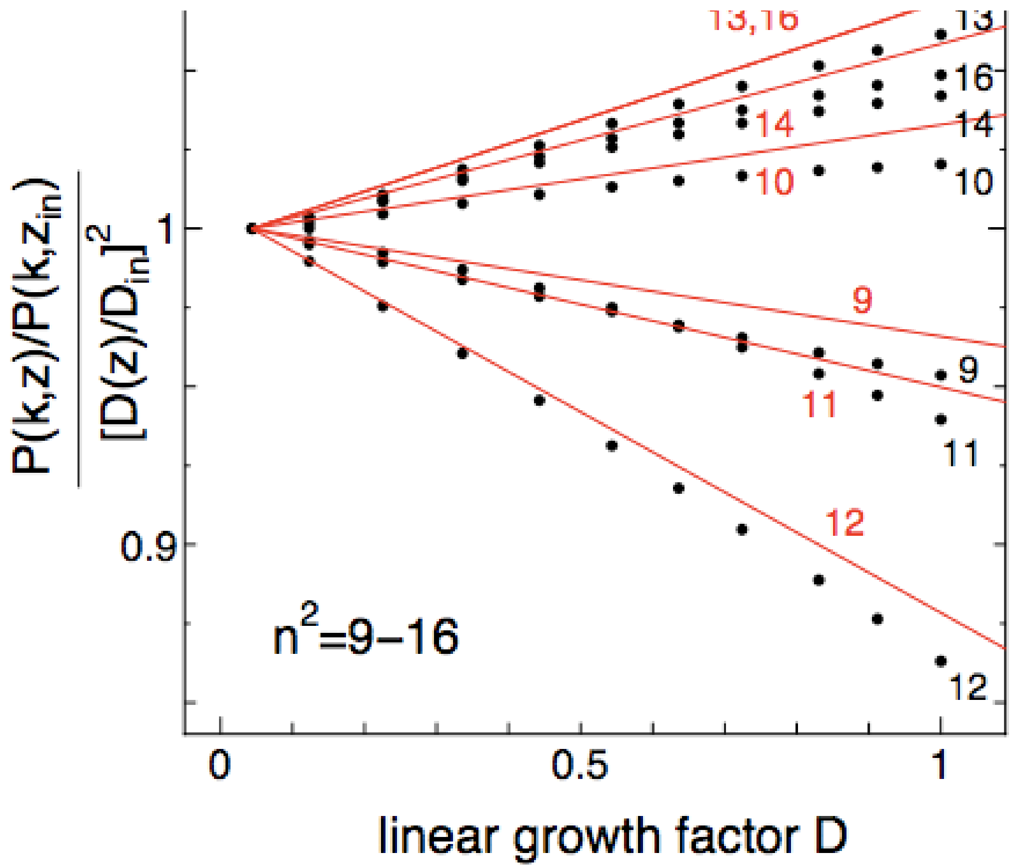


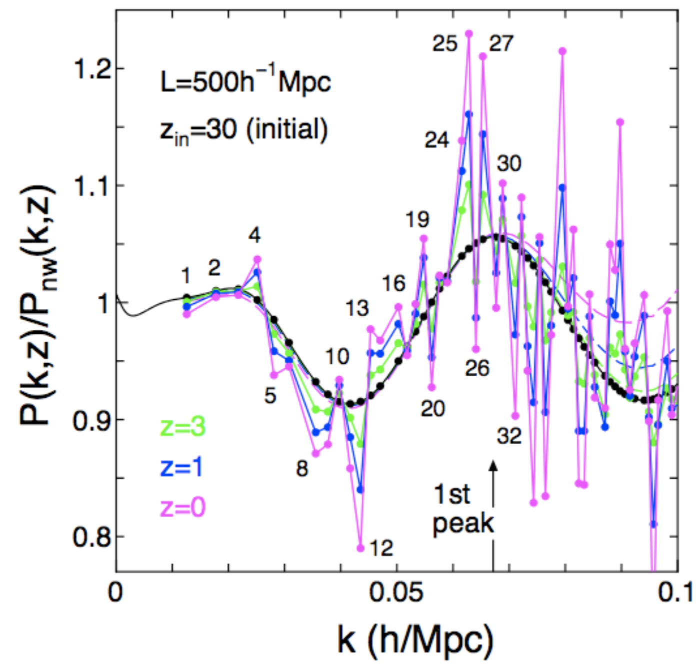
HOWEVER, it does not vanish if the number of modes is finite.

There are only three fundamental modes In a given volume.

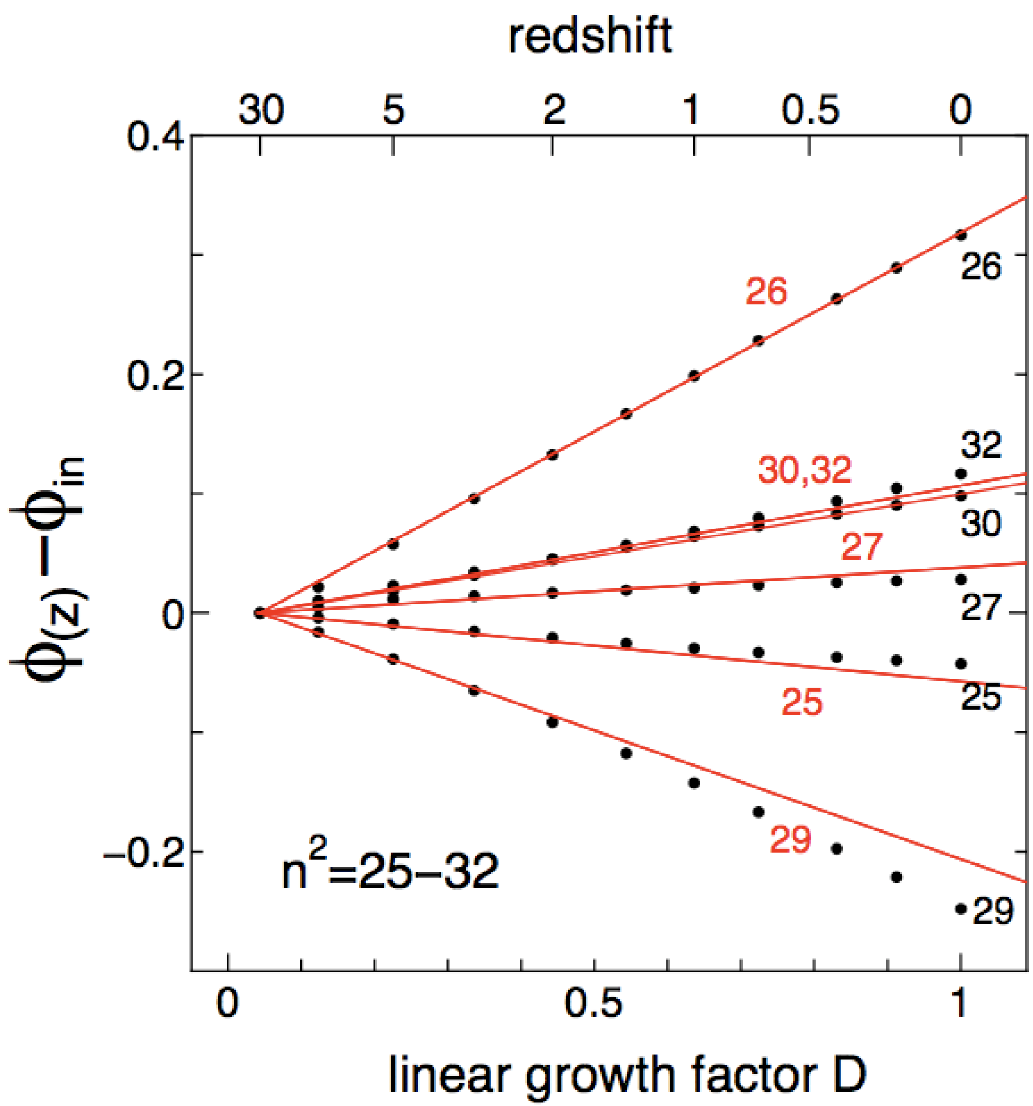
Analytic model prediction

For a given initial density field,
we are able to predict which modes grow
faster / slower than linear theory prediction,
and also how fast / slow.





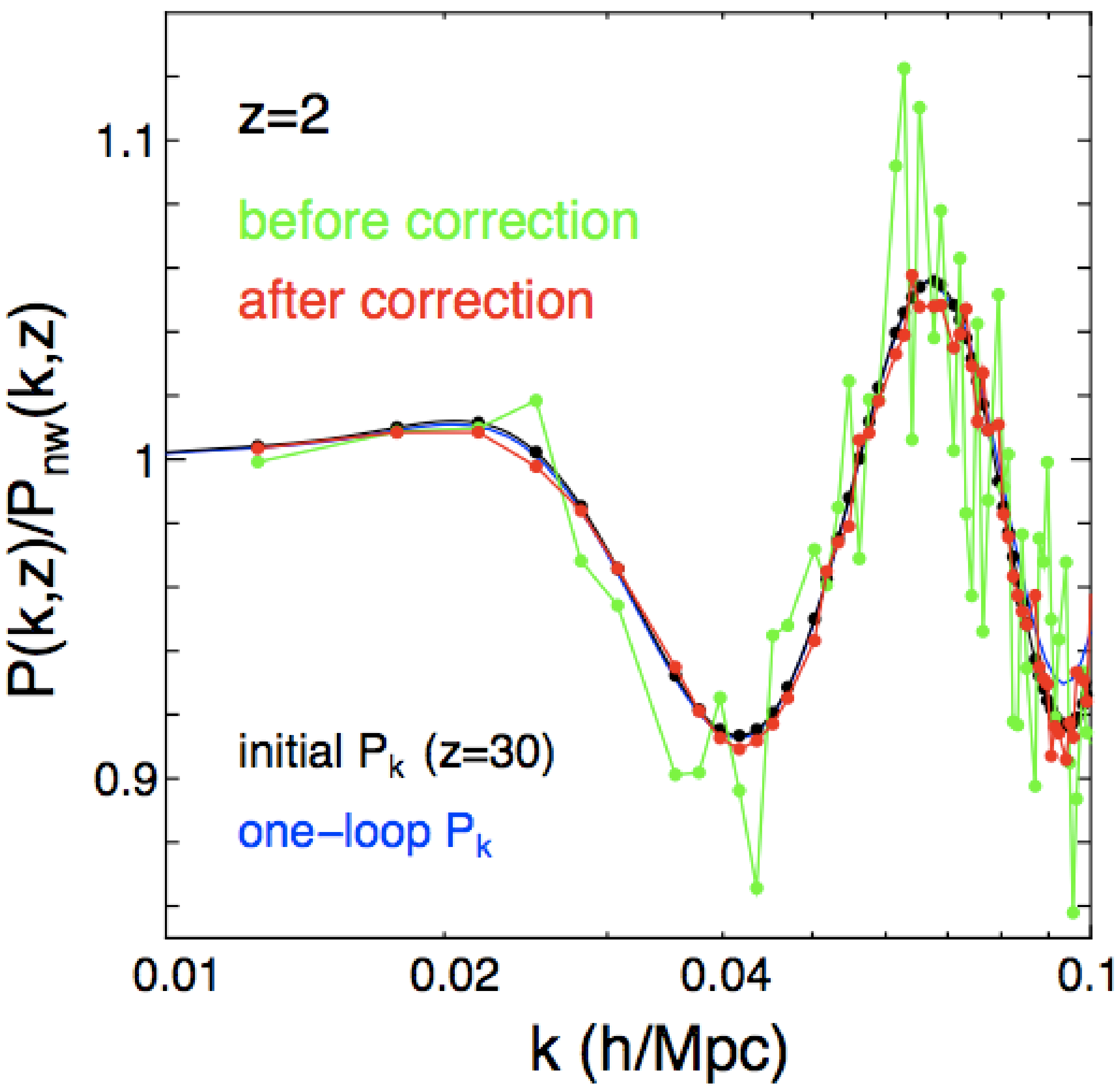
Phase evolution



We can even predict accurately the evolution of phases in Fourier space.

Not sure how useful phase informations are…

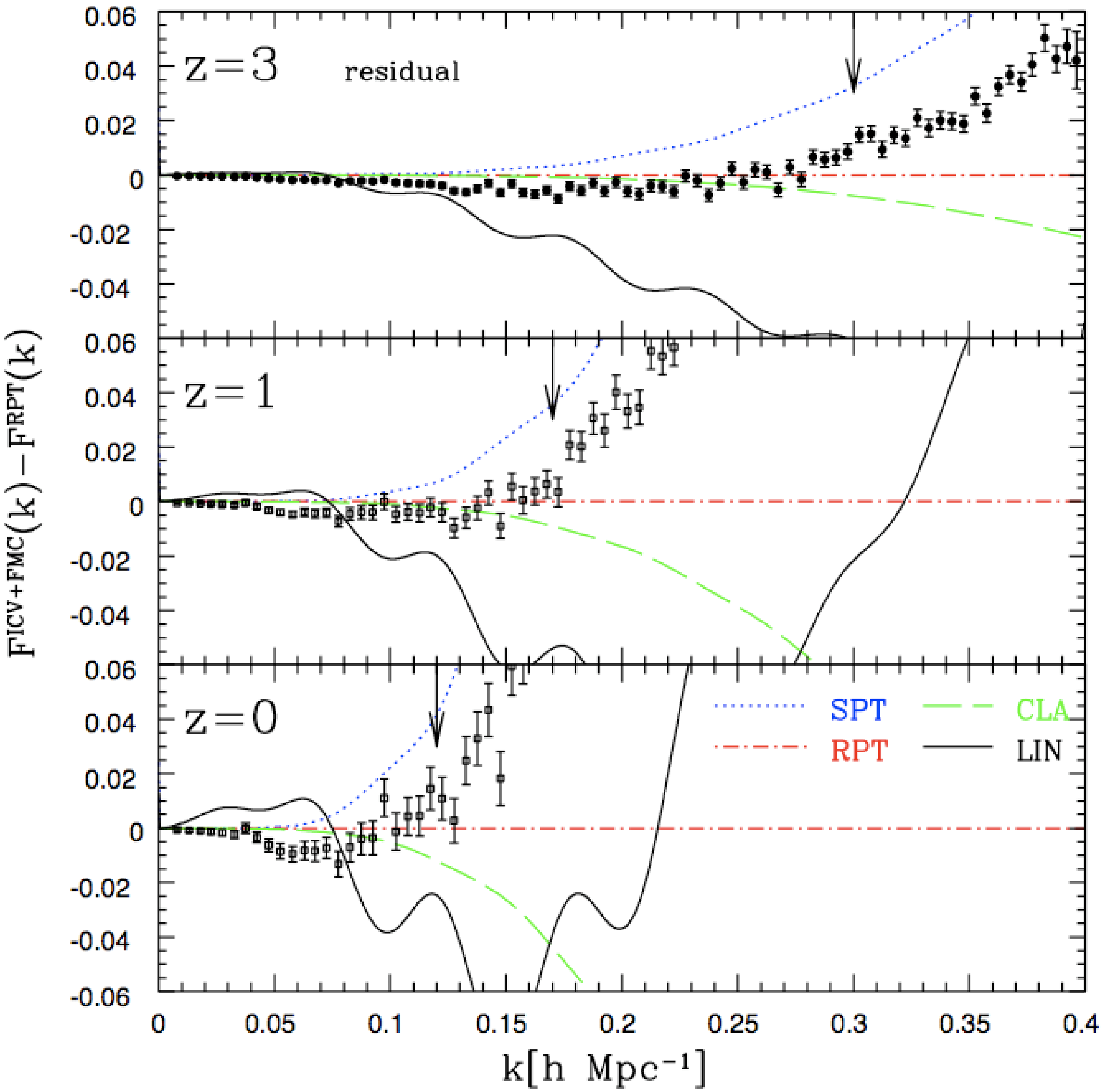
P(k) correction



The finite-mode coupling explains most of the deviations from LT. (See green then red)

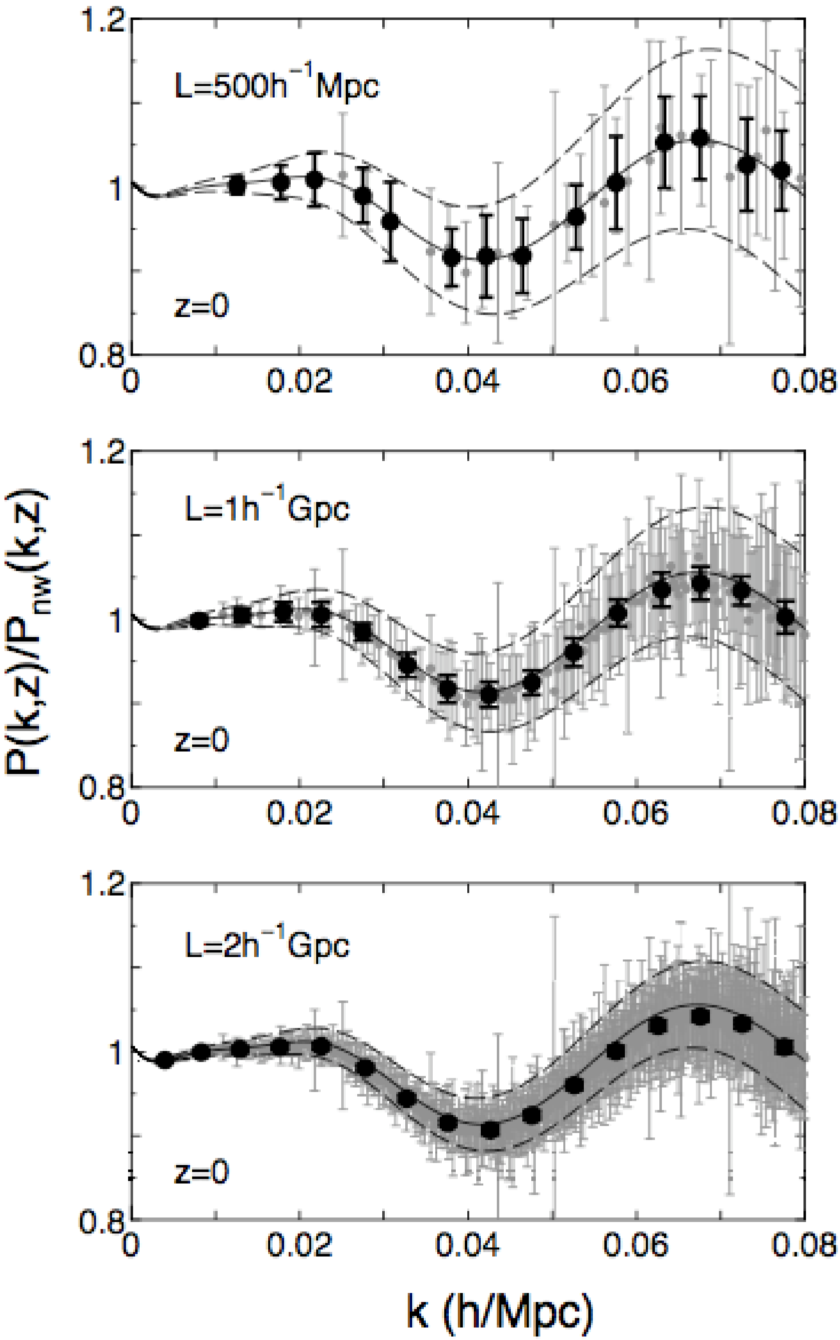
Slight deviation at the 1-st peak is real. (Wang & Szalay)

“Consistent” model predictions



After correcting all of the known effects, theoretical model predictions agree at this level!

Nishimich+, in prep.



So far so good.

But dispersion still remains...

100 realizations for boxsizes

0.5, 1, 2 Gpc.

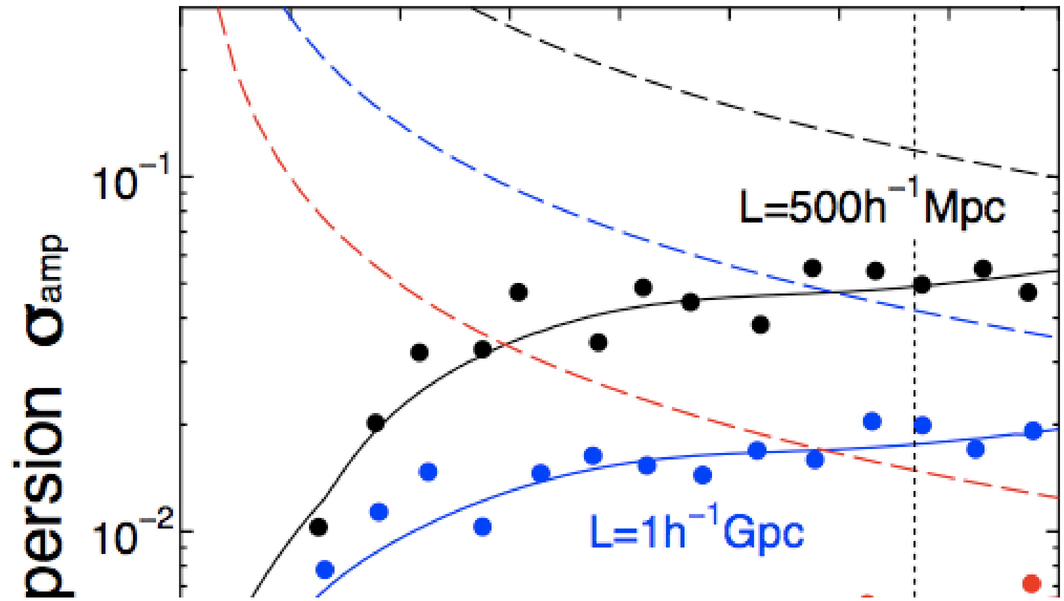
Dispersions in P(k) are as large as

~ 10 % for 0.5 Gpc box

~ a few % for 1 Gpc

~ 1 % for 2 Gpc.

Dispersions as large as intrinsic Gaussian scatter



Symbols: this study

Dashed lines:

random Gaussian

dispersions $= \sqrt{2/N_k}$

Dispersion from the linear theory

$$\sigma^2_{\rm amp}(k,z) = \frac{4P_{22}(k,z)}{P_{11}(k,z)}\frac{1}{\Delta N_k} \approx 2\% \left(\frac{L}{1{\rm Gpc}/h}\right)^{-3/2}\left(\frac{\Delta k}{0.005h/{\rm Mpc}}\right)^{-1/2}$$

Summary

Nearly all the features in the power spectra at large-scales are well-understood.
e.g.) initial random scatter, small wiggles

Dispersion owing to the finite-mode coupling is

$$\approx 2\% \left(\frac{L}{1\mathrm{Gpc}/h} \right)^{-3/2} \left(\frac{\Delta k}{0.005h/\mathrm{Mpc}} \right)^{-1/2} \quad \text{at } z = 0$$

★ Use large-box (> 1Gpc) simulations to get accurate statistics.

Covariance matrix of power spectrum

Covariance for wavenumbers k1, k2:

$$\mathrm{cov}(k_1,k_2) = \left\langle \left[P(k_1) - \langle P(k_1)\rangle\right]\left[P(k_2) - \langle P(k_2)\rangle\right] \right\rangle$$

$$= \frac{2}{N_k} P^2(k)\delta_{k_1,k_2} + T(k_1,k_2)$$

(Nk：number of modes)

1st term >> 2nd term at (quasi)linear scale

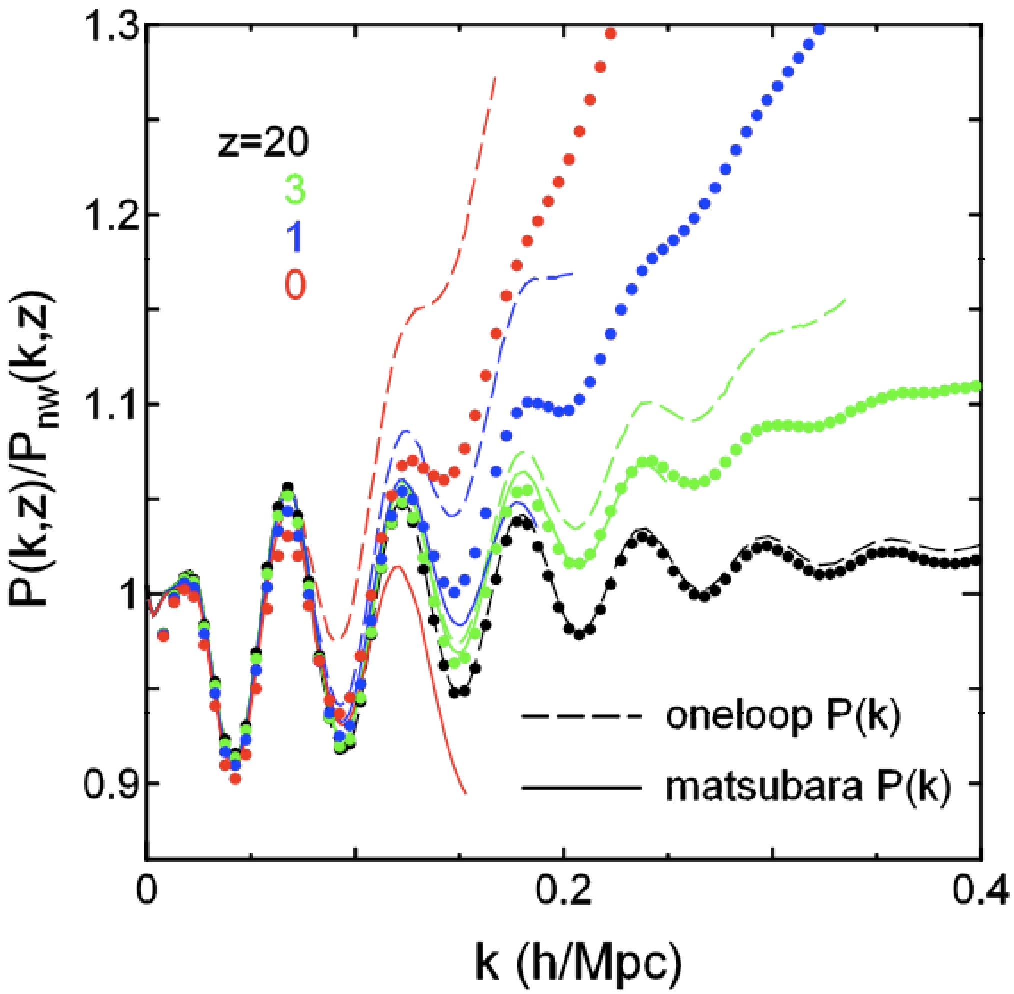
Diagonal part

$$\mathrm{cov}(k_1,k_1) = \sigma_p^2 = \left\langle \left[P(k_1) - \langle P(k_1)\rangle\right]^2 \right\rangle$$

(Scoccimarro, Zaldarriaga & Hui 1999)

Brute-force PM simulations

1 Gpc/h box, 256^3 particles and meshes



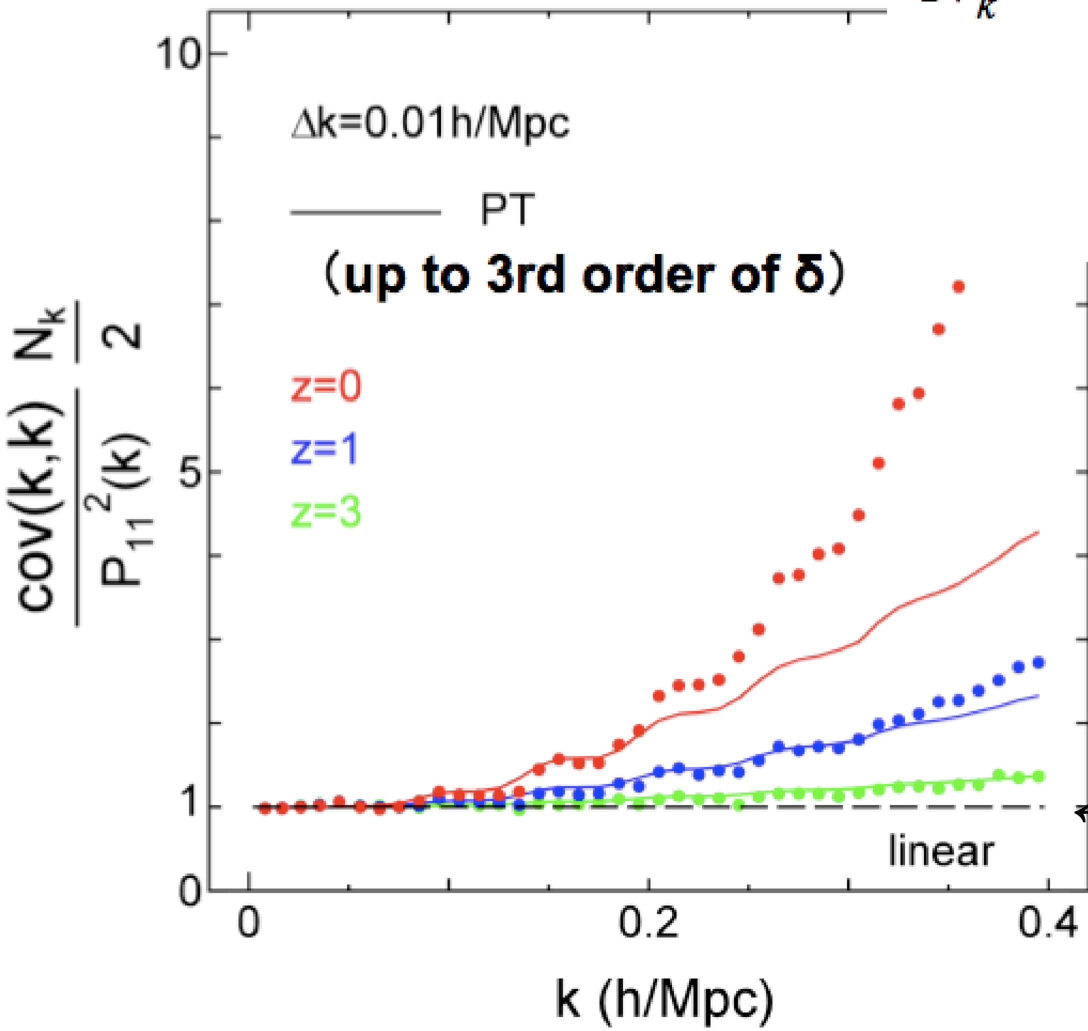
2000 realizations

(4 minutes per 1 run)

Diagonal elements

$$\frac{2}{N_k}P^2 = \frac{2}{N_k}\left(P_{11} + P_{22} + P_{13}\right)^2$$

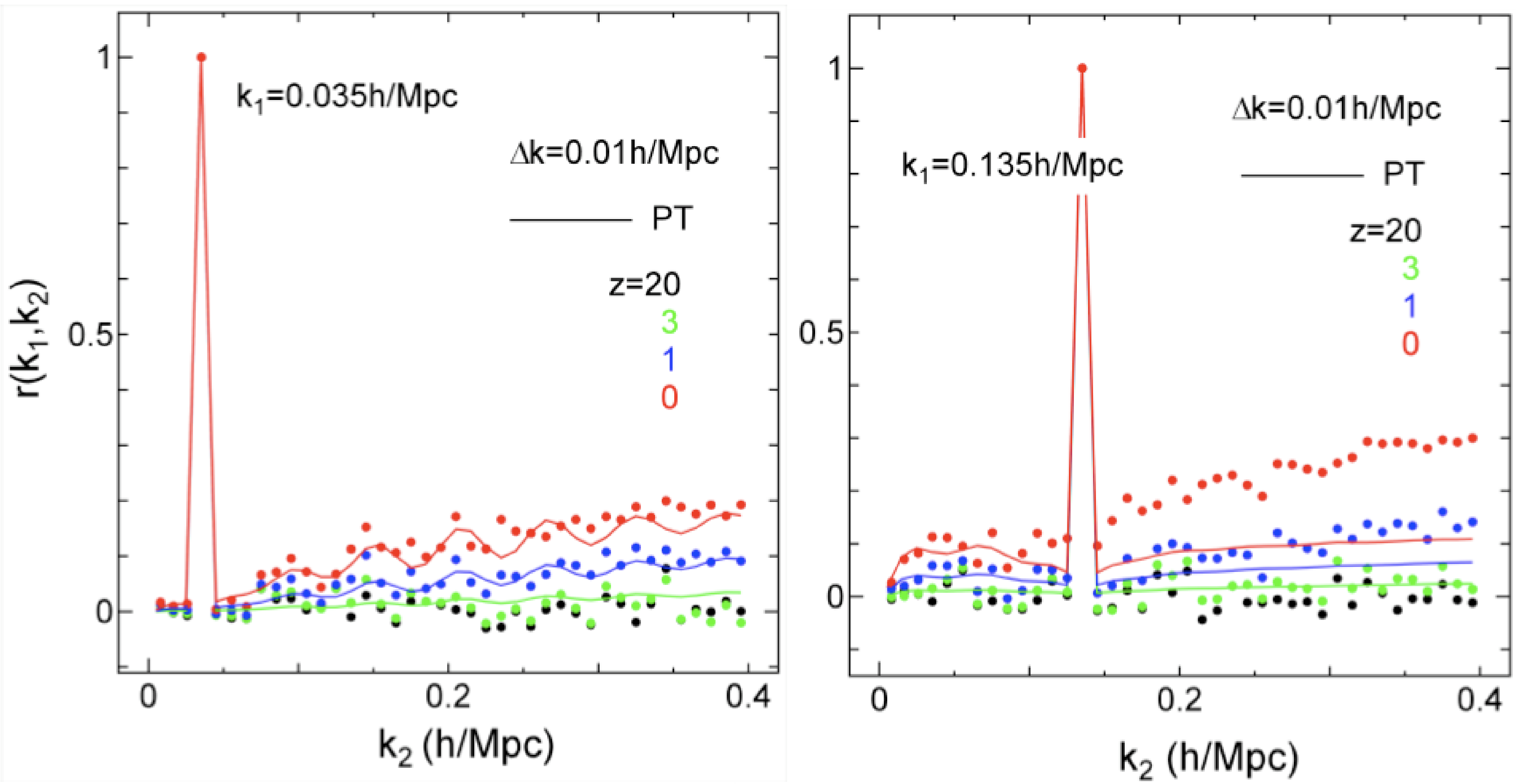
$$\cong \frac{2}{N_k}P_{11}^2 + \frac{4}{N_k}P_{11}\left(P_{22} + P_{13}\right)$$



←*Linear theory prediction*

Off-diagonal elements

$$r(k_1,k_2)=\frac{\mathrm{cov}(k_1,k_2)}{\sqrt{\mathrm{cov}(k_1,k_1)\times \mathrm{cov}(k_2,k_2)}}$$



Takahashi et al. in prep.

Summary

Nearly all the features in the power spectra at large-scales are well-understood.
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Dispersion owing to the finite-mode coupling is

$$\approx 2\% \left(\frac{L}{1\mathrm{Gpc}/h} \right)^{-3/2} \left(\frac{\Delta k}{0.005h/\mathrm{Mpc}} \right)^{-1/2} \quad \text{at } z = 0$$

★ Use large-box (> 1Gpc) simulations to get accurate statistics.

Summary 2

Matter power spectrum at large-scales under control. Further calibration of analytic models.

Large volume (> 1-2Gpc) simulations necessary.

Covariance matrices computed using a sufficiently large number of realizations. Further analysis (contributions from non-linear, numerical effects) needed.

Next step: Halo power spectrum, covariance mat.