Cosmological power spectra in a closure theory

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Goal of this work

…is to validate a new method for accurate prediction of the power spectrum based on the *closure approximation* [Atsushi Taruya's talk and see ApJ674(2008)617 or arxiv:0708.1367] , in which the spectrum is described by a set of non-linear integro-differential equations.

In this poster, we numerically solve them without any extra approximations.

Closure approximation (abstract)

- Eular equation and continuity equation for each order of moments require information about higher-order moments.

  = BBGKY hierarchy (in principle, we cannot get a closed set of equations)

- The *direct interaction approximation* is frequently used for the turbulence theory, which makes a set of equations being closed by neglecting some kinds of diagrams providing subdominant contributions. This approximation is identical to a mathematical technique named the reversed expansion.

  → Implemented this to the 1-loop perturbation theory, we have obtained a closed set of evolution equations for the propagator G_{ab} and density/velocity power spectra R_{ab} (and P_{ab}), taking into account up to 1-loop diagrams.

Details can be seen in Taruya's talk.

Re-arrangement of equations

- More symmetric forms

$$\Lambda_{ab} G_{bc}(|\boldsymbol{k}|;\eta,\eta') = \int_{\eta'}^{\eta} d\eta'' M_{as}(|\boldsymbol{k}|;\eta,\eta'') G_{sc}(|\boldsymbol{k}|;\eta'',\eta')$$

$$\Lambda_{ab} R_{bc}(|\boldsymbol{k}|;\eta,\eta') = \int_{\eta_0}^{\eta} d\eta'' M_{as}(|\boldsymbol{k}|;\eta,\eta'') R_{\overline{sc}}(|\boldsymbol{k}|;\eta'',\eta')$$

$$+ \int_{\eta_0}^{\eta'} d\eta'' N_{al}(|\boldsymbol{k}|;\eta,\eta'') G_{cl}(|\boldsymbol{k}|;\eta',\eta'')$$

$$\Sigma_{abcd} P_{cd}(|\boldsymbol{k}|;\eta) = \int_{\eta_0}^{\eta} d\eta'' M_{bs}(|\boldsymbol{k}|;\eta,\eta'') R_{as}(|\boldsymbol{k}|;\eta,\eta'')$$

$$+ \int_{\eta_0}^{\eta} d\eta'' N_{bl}(|\boldsymbol{k}|;\eta,\eta'') G_{al}(|\boldsymbol{k}|;\eta,\eta'') + (a \leftrightarrow b)$$

$$\eta = \log D(z) \qquad \Lambda_{ab} = \delta_{ab}\frac{\partial}{\partial \eta} + \Omega_{ab}(\eta)$$

Re-arrangement of equations

- Left-hand side

$$\Omega_{ab}(\eta) = \begin{pmatrix} 0 & -1 \\ -\dfrac{3}{2f^2}\Omega_m & \dfrac{3}{2f^2}\Omega_m - 1 \end{pmatrix}$$

The explicit forms of $\Omega_{2i}(\eta)$ depend on the Poisson equation and the Friedmann equation. This is a case with the Newtonian gravity and the LCDM model.

- Integration by wave-number

$$M_{as}(|\boldsymbol{k}|;\eta,\eta'') = 4\int \frac{d^3k'}{(2\pi)^3} \gamma_{apq}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}')\gamma_{lrs}(\boldsymbol{k}'-\boldsymbol{k},\boldsymbol{k}) \times G_{ql}(|\boldsymbol{k}'|,\eta,\eta'')R_{pr}(|\boldsymbol{k}-\boldsymbol{k}'|;\eta,\eta'')$$

$$N_{al}(|\boldsymbol{k}|;\eta,\eta'') = 2\int \frac{d^3k'}{(2\pi)^3} \gamma_{apq}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}')\gamma_{lrs}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}') \times R_{qs}(|\boldsymbol{k}'|,\eta,\eta'')R_{pr}(|\boldsymbol{k}-\boldsymbol{k}'|;\eta,\eta'')$$

Among 8 components of γ_{apq}, only γ_{112} γ_{121} γ_{222} have non-zero value.

Strategy for numerics

- Expand G_{ab} and R_{ab} by a set of basis functions of k.

$$G_{ab}(\eta,k)=\sum G_{ab,m}(\eta)T_m(k)$$

- Replacing the differential operator by the central difference.

$$\left.\frac{\partial g}{\partial \eta}\right|_{\eta=\eta_n} \approx \frac{g^{(n+1)}-g^{(n-1)}}{2\Delta\eta}$$

- Apply the trapezoidal rule to the time-integrations.

$$\int g(\eta)d\eta \approx \frac{\Delta\eta}{2}\sum(g^{(n)}+g^{(n+1)})$$

- Sequentially solve the recursive equations.

Various treatments of non-linear terms

Lv. 5 : Full non-linear treatment

- Solve the equations without any modifications.

Lv. 4 : R-linearised treatment

- All R_{ab}'s in the right-hand sides are replaced.

Lv. 3 : Semi-linearised treatment

- G_{ab} and R_{ab} in M_{ab} and N_{ab} are replaced by those calculated in the linear theory

Lv. 2 : Linearised treatment

- All G_{ab}'s and R_{ab}'s are replaced by linear ones.

Lv. 1 : 0-th order theory

- All right-hand sides are dropped

These treatments are prepared as references of 'Full' treatment

Lv.1 : 0-th order theory

- Dropped all right-hand side terms

$$\Lambda_{ab} G^0_{bc}(|\boldsymbol{k}|;\eta,\eta') = 0$$

$$\Lambda_{ab} R^0_{bc}(|\boldsymbol{k}|;\eta,\eta') = 0$$

In the case of the Einstein-de Sitter universe, these equations can be analytically solved :

$$G^0_{ab}(|\boldsymbol{k}|;\eta,\eta') = \frac{\Theta(\eta-\eta')}{5}\left[e^{(\eta-\eta')}\begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix} + e^{-(3/2)(\eta-\eta')}\begin{pmatrix} 2 & -2 \\ -3 & 3 \end{pmatrix} \right]$$

As for the power spectrum, we focus on the contribution from the growing mode :

$$R^0_{sc}(|\boldsymbol{k}|;\eta,\eta'') = e^{\eta+\eta''} P_0(k) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

where $P_0(k)$ is the linearly extrapolated power spectrum at the present time.

Lv.2: Linearised treatment

- Replacing all right-hand side quantities by those in the linear theory

$$\Lambda_{ab} G^L_{bc}(|\boldsymbol{k}|;\eta,\eta') = \int_{\eta'}^{\eta} d\eta'' M^L_{as}(|\boldsymbol{k}|;\eta,\eta'') G^0_{sc}(\eta'',\eta')$$

$$\Lambda_{ab} R^L_{bc}(|\boldsymbol{k}|;\eta,\eta') = \int_{\eta_0}^{\eta} d\eta'' M^L_{as}(|\boldsymbol{k}|;\eta,\eta'') R^0_{\underline{sc}}(|\boldsymbol{k}|;\eta'',\eta') + \int_{\eta_0}^{\eta'} d\eta'' N^L_{al}(|\boldsymbol{k}|;\eta,\eta'') G^0_{cl}(|\boldsymbol{k}|;\eta',\eta'')$$

Lv.2: Linearised treatment

- Linearised kernels

$$M^{L}_{as}(|\boldsymbol{k}|;\eta,\eta'') = 4\int \frac{d^3k'}{(2\pi)^3}\gamma_{apq}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}')\gamma_{lrs}(\boldsymbol{k}'-\boldsymbol{k},\boldsymbol{k}) \times G^{0}_{ql}(|\boldsymbol{k}'|,\eta,\eta'')R^{0}_{pr}(|\boldsymbol{k}-\boldsymbol{k}'|;\eta,\eta'')$$

$$N^{L}_{al}(|\boldsymbol{k}|;\eta,\eta'') = 2\int \frac{d^3k'}{(2\pi)^3}\gamma_{apq}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}')\gamma_{lrs}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}') \times R^{0}_{qs}(|\boldsymbol{k}'|,\eta,\eta'')R^{0}_{pr}(|\boldsymbol{k}-\boldsymbol{k}'|;\eta,\eta'')$$

The solution should be consistent to the predictions of 1-loop standard perturbation theory (SPT)

$$G^{L}_{ab} = G^{SPT}_{ab} \qquad P^{L}_{ab} = P^{SPT}_{ab} = e^{2\eta}P_0 + e^{4\eta}(P_{13}+P_{22})$$

Lv.3 : Semi-linearised treatment

- The kernels are replaced by linearised ones

$$\Lambda_{ab} G^S_{bc}(|\boldsymbol{k}|;\eta,\eta') = \int_{\eta'}^{\eta} d\eta'' M^L_{as}(|\boldsymbol{k}|;\eta,\eta'') G^S_{sc}(\eta'',\eta')$$

$$\Lambda_{ab} R^S_{bc}(|\boldsymbol{k}|;\eta,\eta') = \int_{\eta_0}^{\eta} d\eta'' M^L_{as}(|\boldsymbol{k}|;\eta,\eta'') R^S_{sc}(|\boldsymbol{k}|;\eta'',\eta') + \int_{\eta_0}^{\eta'} d\eta'' N^L_{al}(|\boldsymbol{k}|;\eta,\eta'') G^S_{cl}(|\boldsymbol{k}|;\eta',\eta'')$$

The difference from Lv.2 is to replace G^0_{ab} by G^S_{ab} itself.

cf. 'steepest descent method' by P.Valageas (2004,2007,2008)

Lv.4 : R-linearised treatment

- Replace all R_{ab}'s in right-hand sides by R_{ab}^{L}

$$\Lambda_{ab} R_{bc}^{R}(|\boldsymbol{k}|;\eta,\eta') = \int_{\eta_0}^{\eta} d\eta'' M_{as}^{R}(|\boldsymbol{k}|;\eta,\eta'') R_{sc}^{L}(|\boldsymbol{k}|;\eta'',\eta') + \int_{\eta_0}^{\eta'} d\eta'' N_{al}^{R}(|\boldsymbol{k}|;\eta,\eta'') G_{cl}^{R}(|\boldsymbol{k}|;\eta',\eta'')$$

$$\Lambda_{ab} G_{bc}^{R}(|\boldsymbol{k}|;\eta,\eta') = \int_{\eta'}^{\eta} d\eta'' M_{as}^{R}(|\boldsymbol{k}|;\eta,\eta'') G_{sc}^{R}(|\boldsymbol{k}|;\eta'',\eta')$$

Lv.4 : R-linearised treatment

- Replace all R's in tight-hand sides by RL

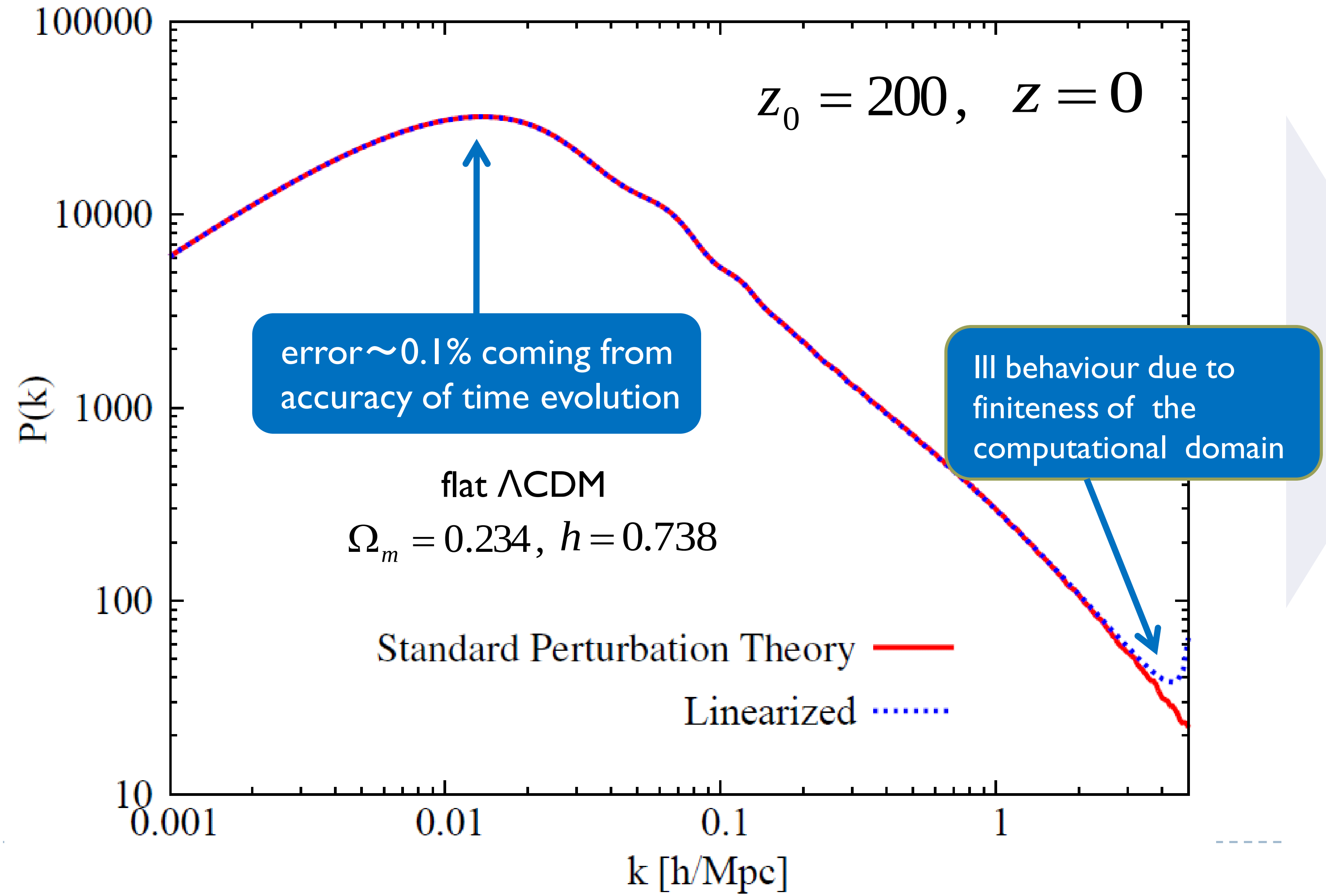
$$M^R_{as}(|\boldsymbol{k}|;\eta,\eta'') = 4\int \frac{d^3k'}{(2\pi)^3}\gamma_{apq}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}')\gamma_{lrs}(\boldsymbol{k}'-\boldsymbol{k},\boldsymbol{k}) \times G^R_{ql}(|\boldsymbol{k}'|,\eta,\eta'') R^L_{pr}(|\boldsymbol{k}-\boldsymbol{k}'|;\eta,\eta'')$$

$$N^R_{al}(|\boldsymbol{k}|;\eta,\eta'') = N^L_{al}(|\boldsymbol{k}|;\eta,\eta'')$$

G^S_{ab} should be consistent to the propagator calculated in the closure theory with the Born approximation

$$G^S_{ab} = G^{CLA}_{ab}$$

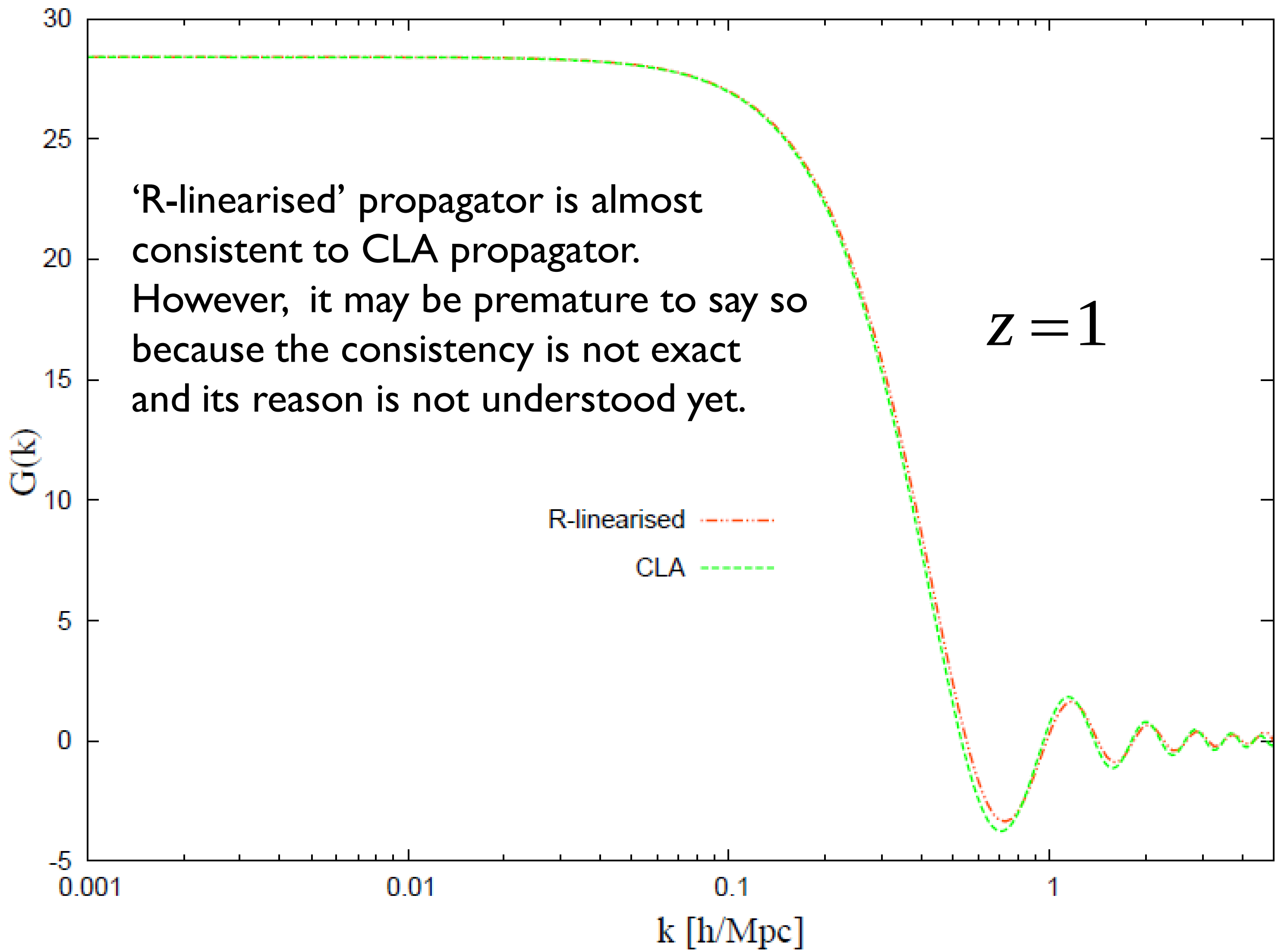
Comparison (Lv.2 vs SPT)



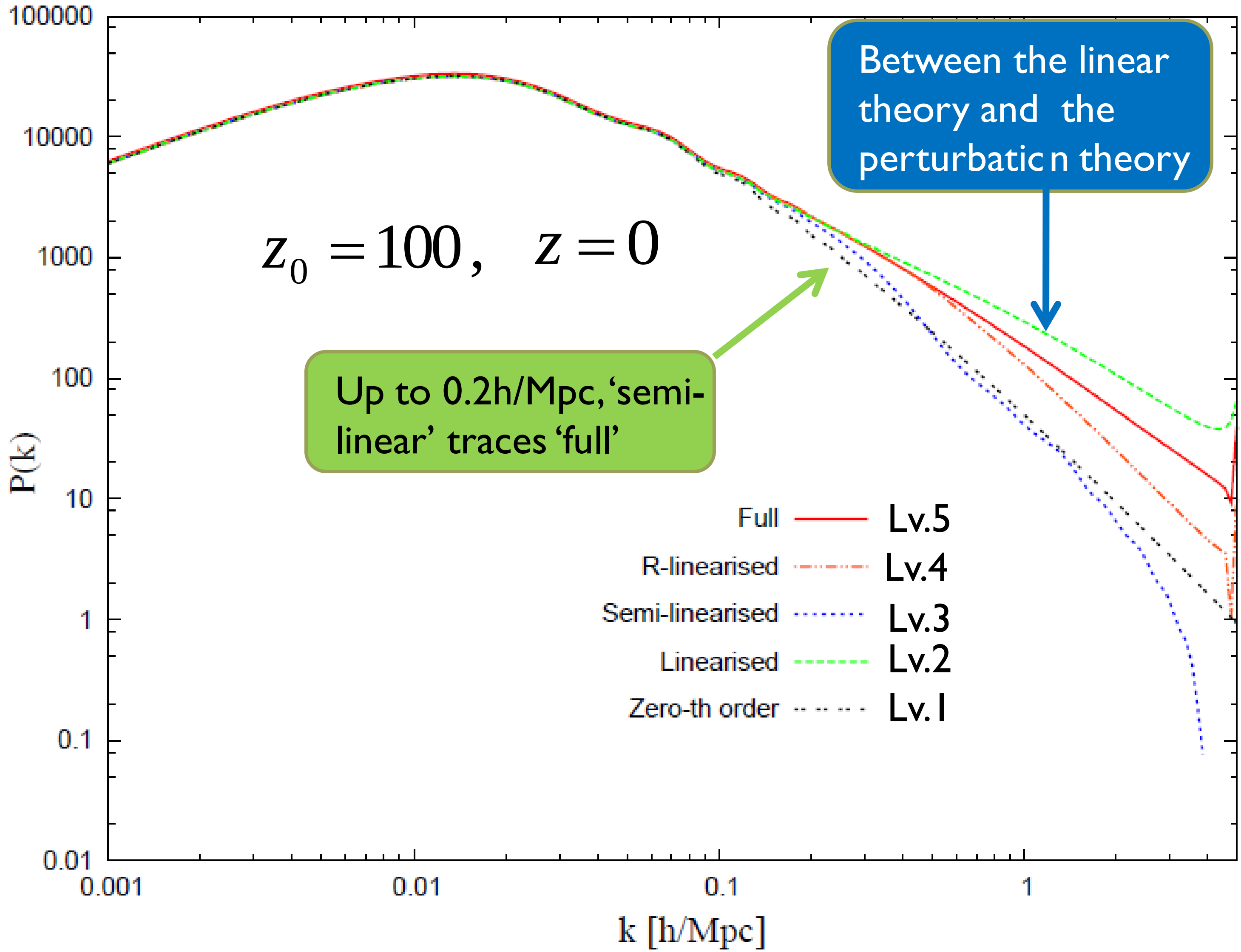
Comparison (Lv.4 vs CLA)

[preliminary]

'R-linearised' propagator is almost consistent to CLA propagator. However, it may be premature to say so because the consistency is not exact and its reason is not understood yet.

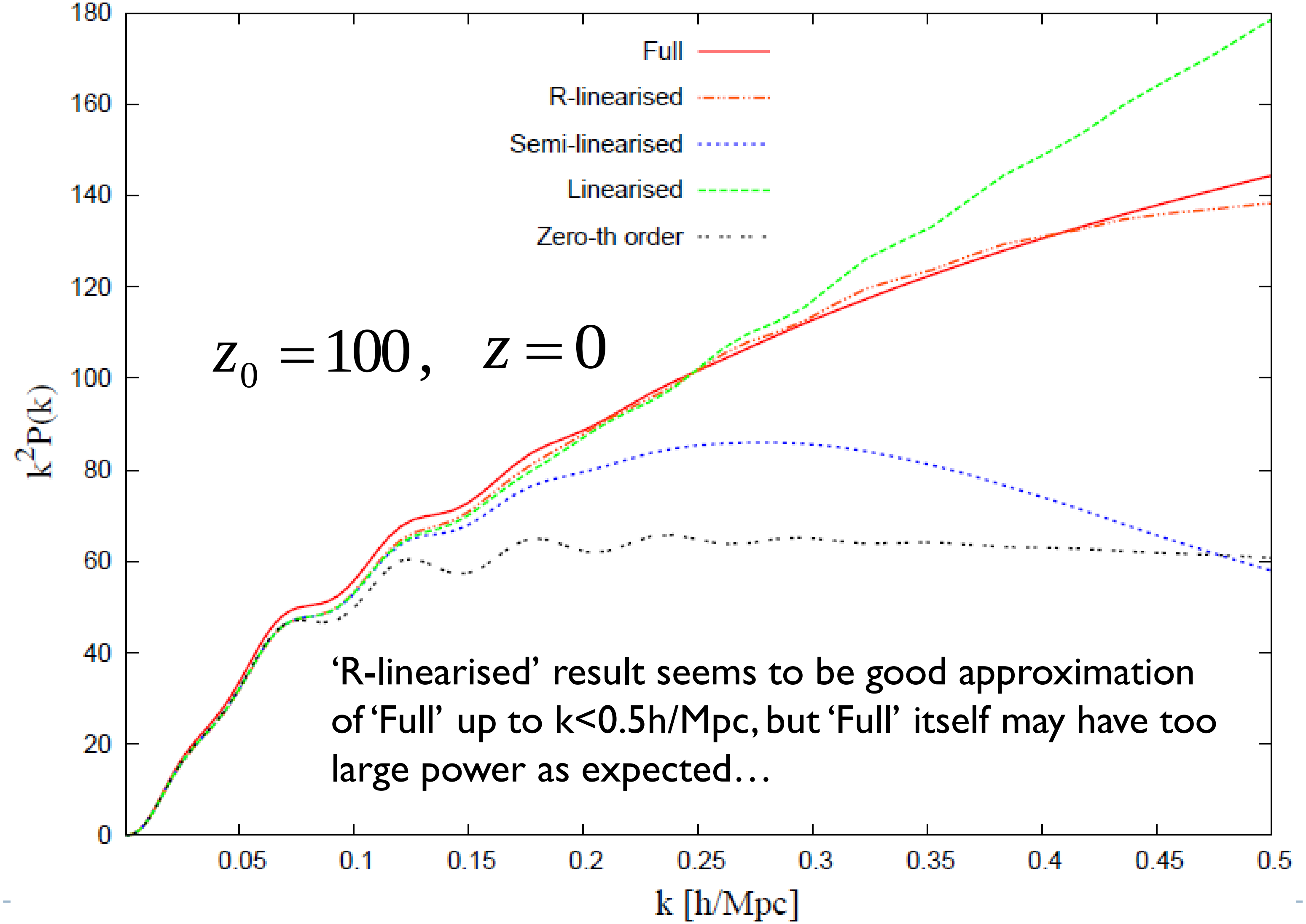


Comparison (variety)

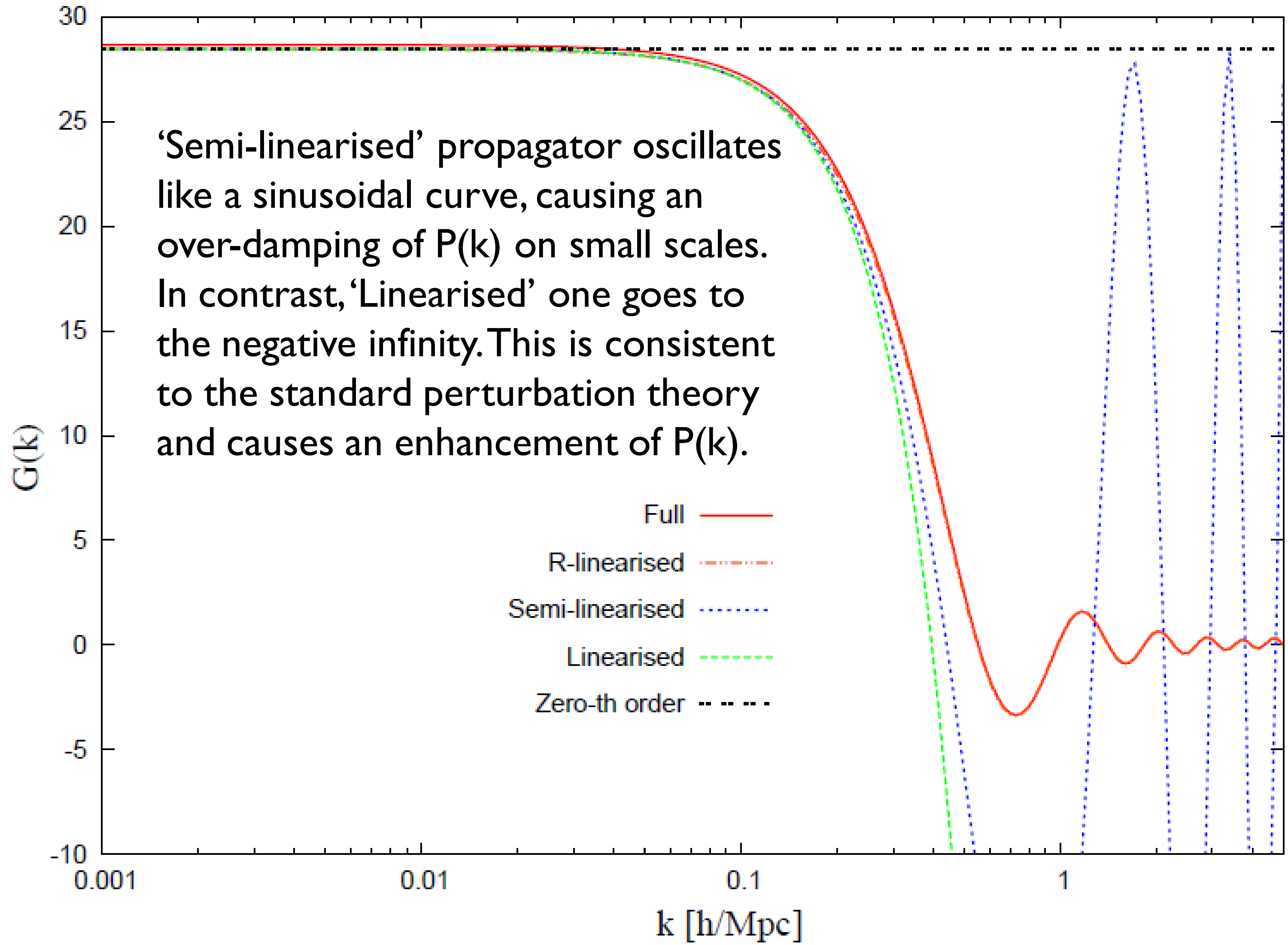


Comparison (detail)

[preliminary]



Comparison (propagator)



Summary

In order to validate our new method for the prediction of the density power spectra based on the closure theory, we directly solve the evolution equations without any extra approximations.

- In Linear Theory (Lv.1), the density power spectrum is dampped for $k > 0.3h/\mathrm{Mpc}$
- In Linearised Source (Lv.2), the spectra coincides with those predicted in SPT.
- In Semi-Linearised (Lv.3), the spectra mimic those for Full (Lv.5) for $k > 0.2h/\mathrm{Mpc}$, while a strong suppression is observed on small scales.
- In R-linearised (Lv.4), this treatment is good approximation for both power spectra and propagators.

future work

- Application to modified gravity theories.
  - Modified gravity theories which modify the Poisson equation can be applied to our scheme. It is realised by only changing the matrix $\Omega_{ab}(\eta)$ in LHS of the evolution equations and/or the vertexes γ_{abc} .

[Hiramatsu, Koyama and Taruya, work in progress]

- Improve the integration scheme to be faster. One candidate is to apply the Gaussian quadrature.
- Make a detailed comparison with N-body simulations.
  - In fact, the non-linearity of 'Lv.5 full' seems to be too strong at this time. The resultant amplitude may be too large in comparison with ones calculated by higher-order Born approximation and N-body simulations. Moreover we observed an anomalous enhancement of the power on large scales at z=0. It is required to check the numerical scheme and understand theoretical reasons.